Zero-Knowledge Proofs And ZK-SNARKs

Foundations Seminar

Helger Lipmaa, April, 2025

Brief Introduction

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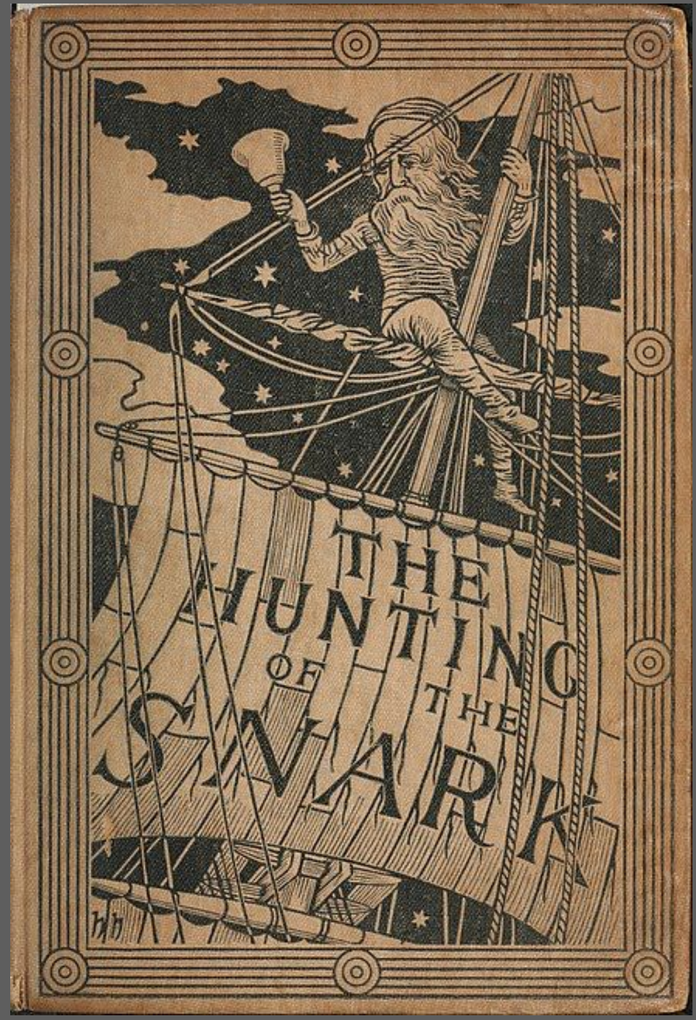
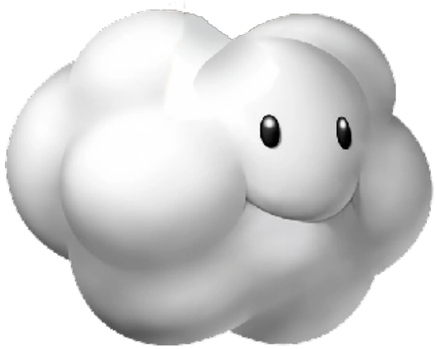
- **Goal:** introduction, trying to sell the hype, collaboration
  - Give an up-to-date overview of the area
  - ZK field is wide, and there is a lot of collaborations possible
  - Coding theory, ML, formal verification, …

ZK-SNARKs: Motivations



Motivation: Verifiable Computation



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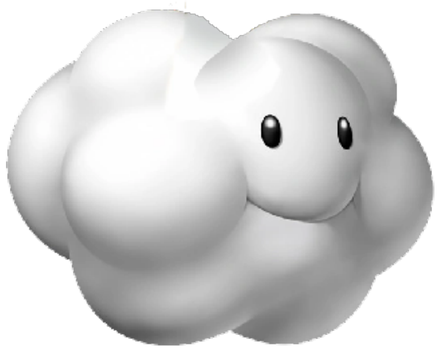

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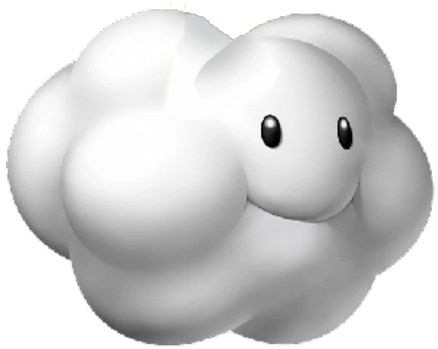

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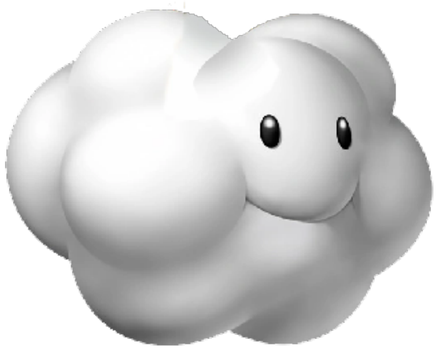

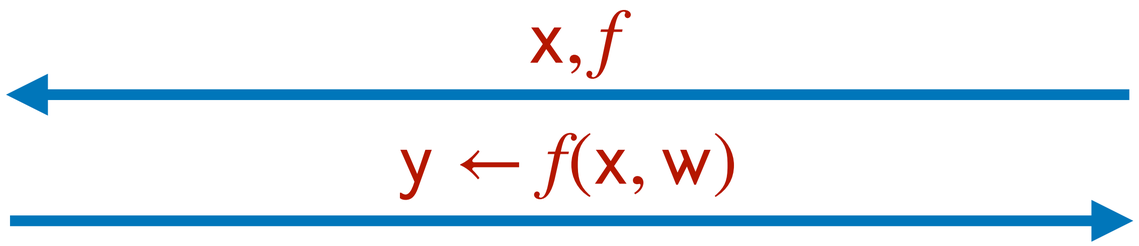

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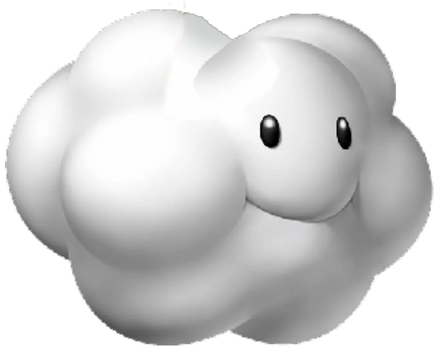

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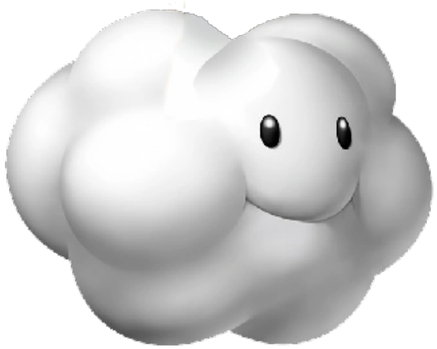

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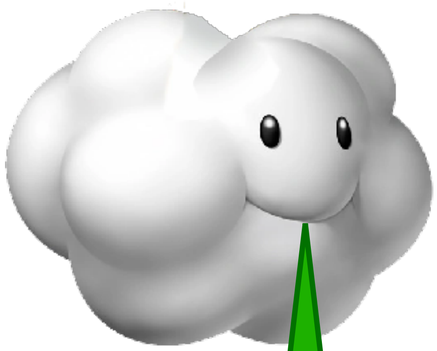

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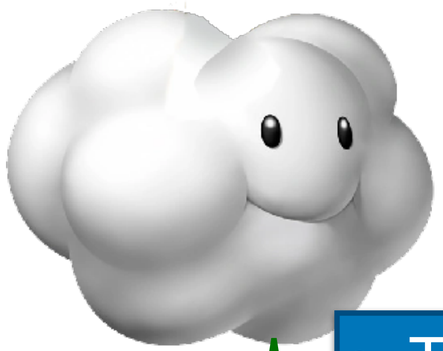

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- The whole **currently** popular ZK field could also be called "verifiable computation" since that is the driving application
- ZK is stuck as the "sexy" name
- ZK has other, more classical applications, like authentication, that currently get less attention
  - Less money ☹…

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Proof can be interactive:
- Consist of several message back and forth

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- **Zero-Knowledge:** nothing about the private input of honest prover is leaked

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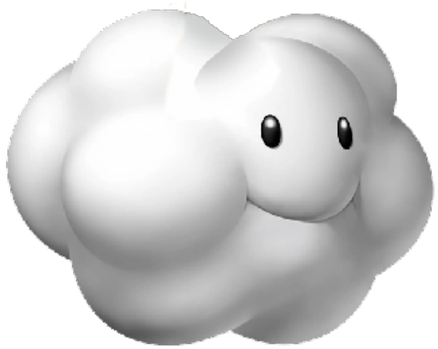

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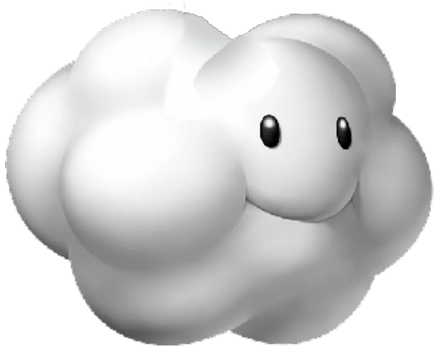

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If verification of a zk-SNARK takes as much time as recomputing, the application is less interesting!

The cloud still preserves her privacy

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- **Scalability**: verifying argument is significantly faster than recomputation
- Important in many applications
  - even **without** privacy… — a lot of what is called ZK is actually **not** ZK but VC

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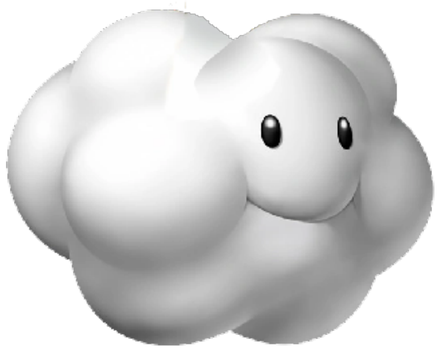

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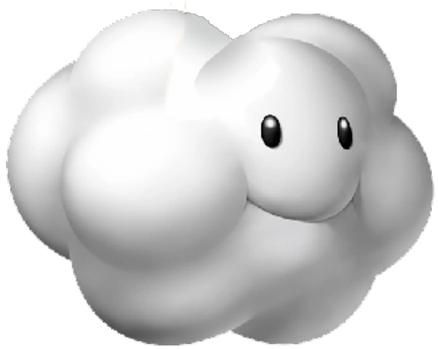

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- **Completeness:** honest verifier accepts if prover **knows** w such that $f(x, w) = y$
- **Knowledge Soundness:** if honest verifier accepts, prover **knows** w
- **Zero-Knowledge: nothing** about w is leaked
- **Efficiency:** verifying π should be **much faster** than redoing the computation

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This number might be outdated

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- **Very active** research topic — prover overhead decreases each year

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Computation f: computing transaction y from public info

Public input: x (public information on blockchain)

Private input: w

- transaction amount, payer account, payee account, …

Computation: f

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Argument π that $f(x, w) = y$

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- The main source of R&D at this moment
- Spending $50B on research can secure $2T money

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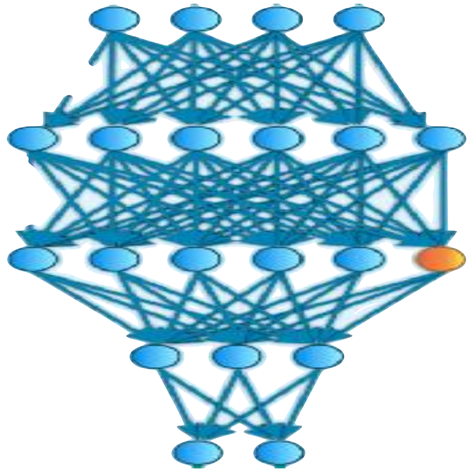

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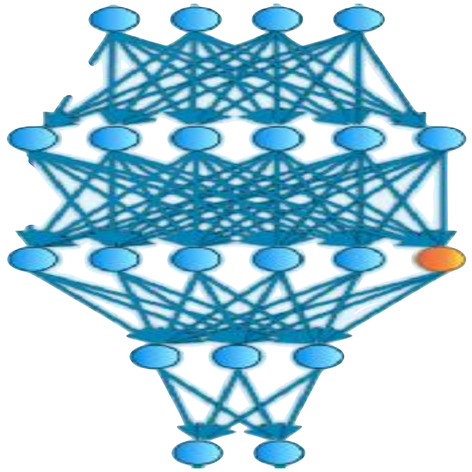

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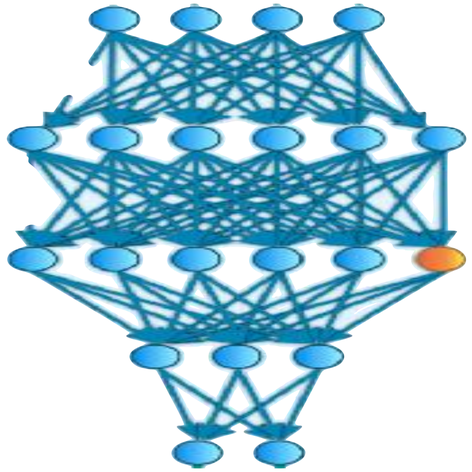

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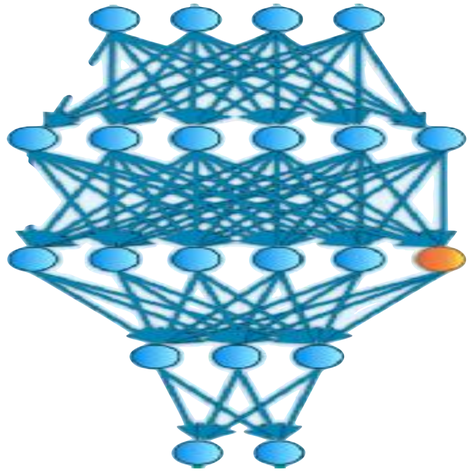

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  - **Property loans, …**

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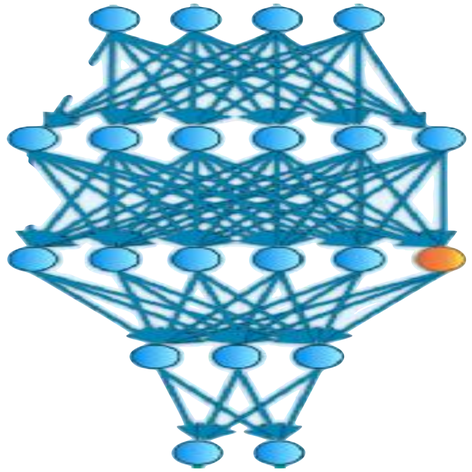
- **Collaboration questions:**
- **In which ML-related questions, ZK can help?**
- **How does the low-level ML computation look like, can it be made more "ZK-friendly"?**



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Computation f: tallying was correct

Public input: x (all incoming signed encrypted ballots)

Private input: w; who voted for who

Computation: f

Public input: x

$\text{true} \leftarrow f(\text{x}, \text{w})$

Argument π that $f(\text{x}, \text{w}) = \text{true}$

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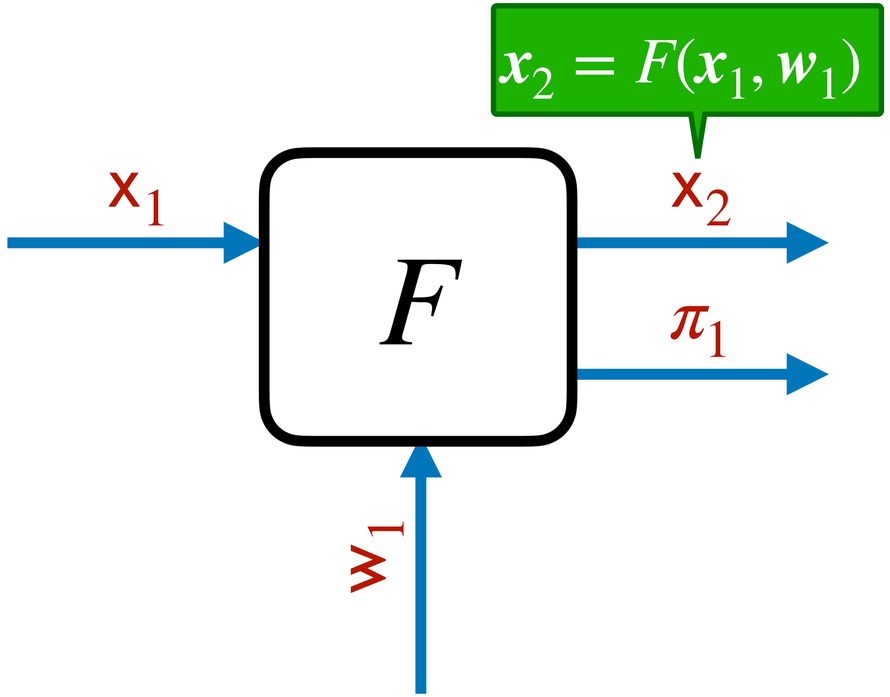
Incrementally Verifiable Computation



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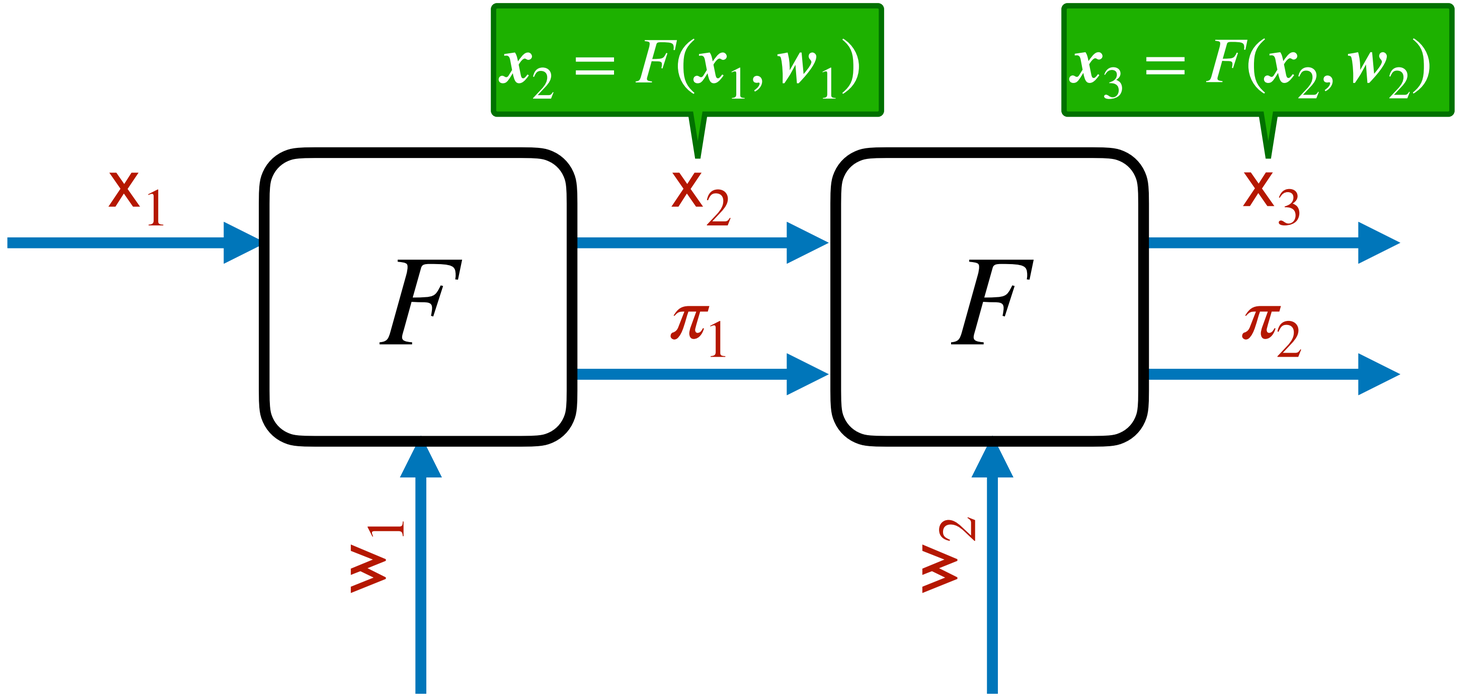

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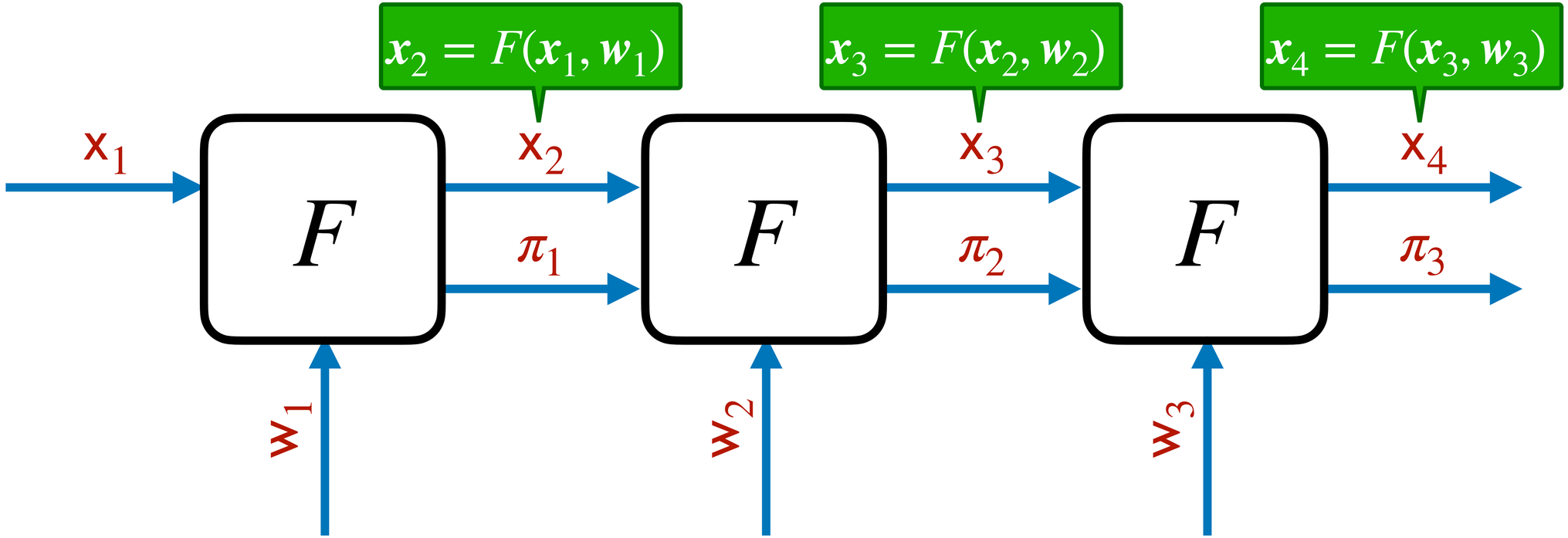

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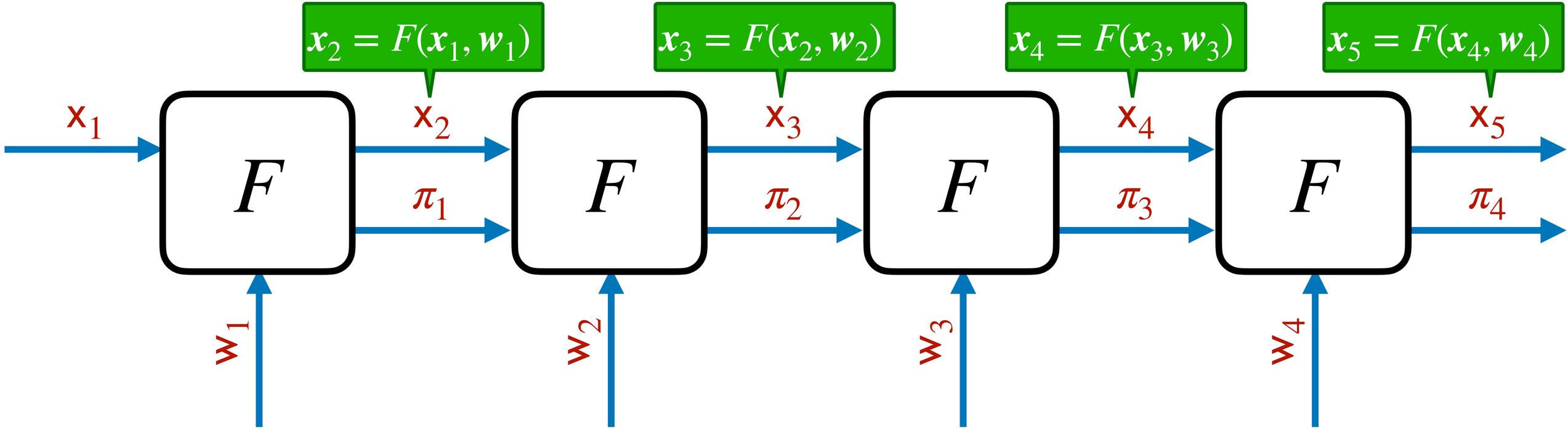

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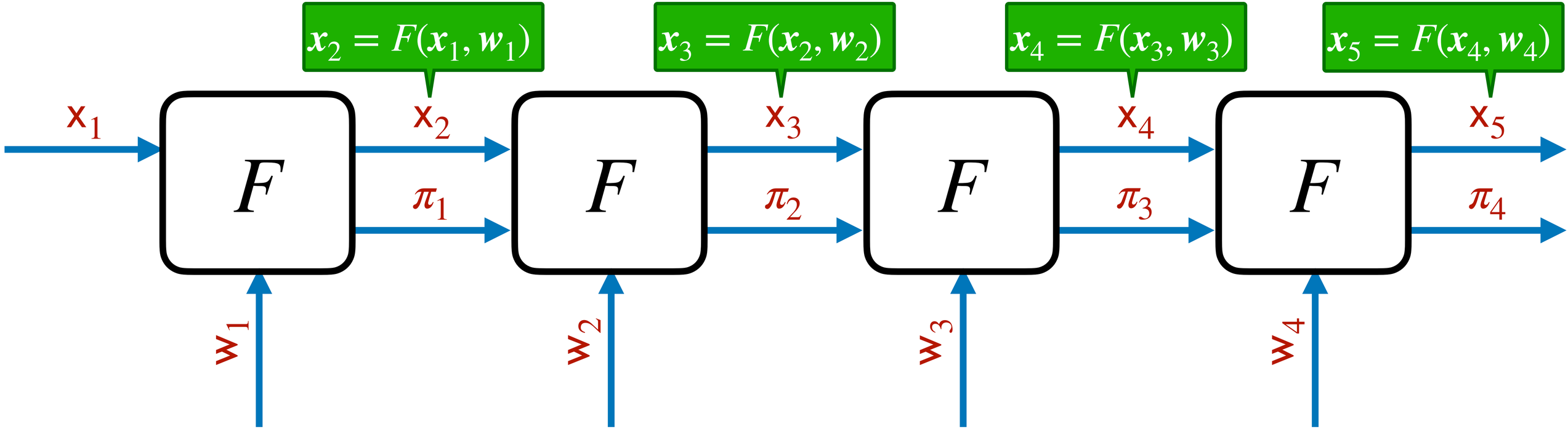

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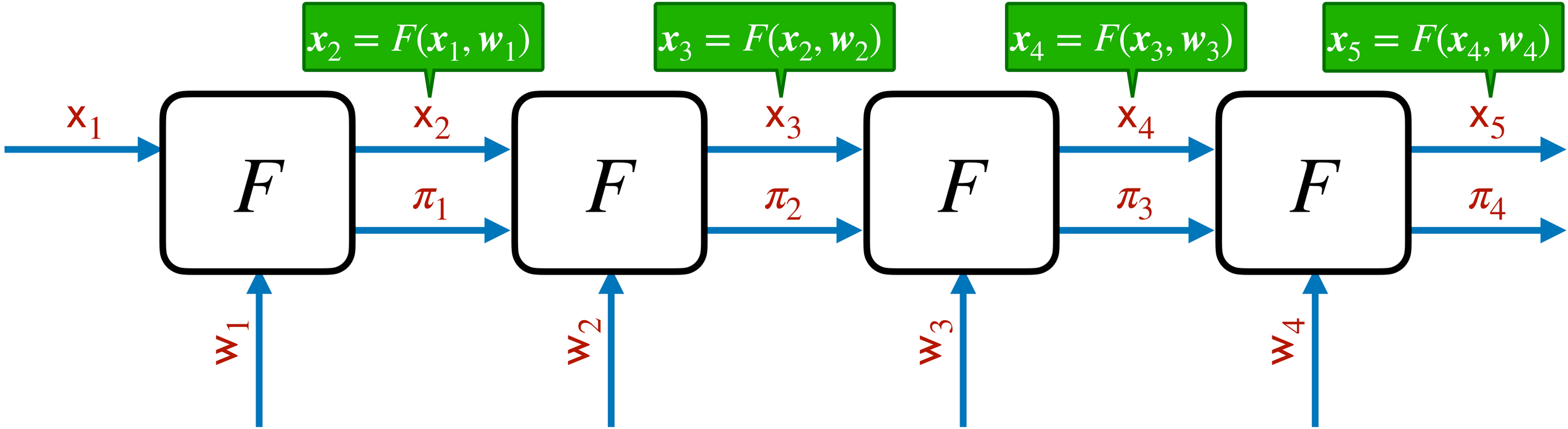

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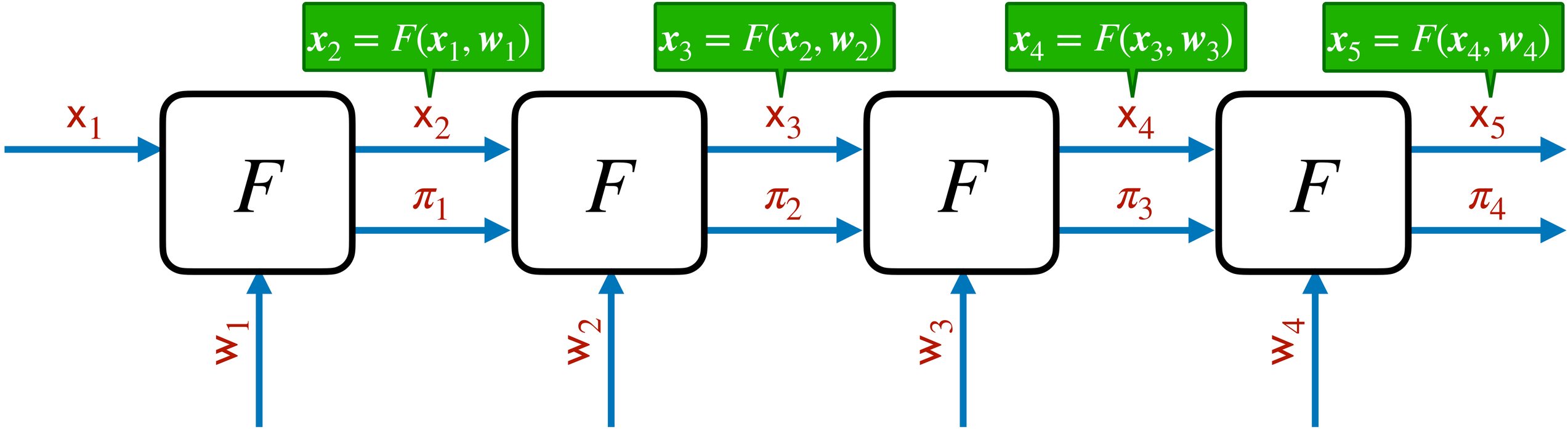

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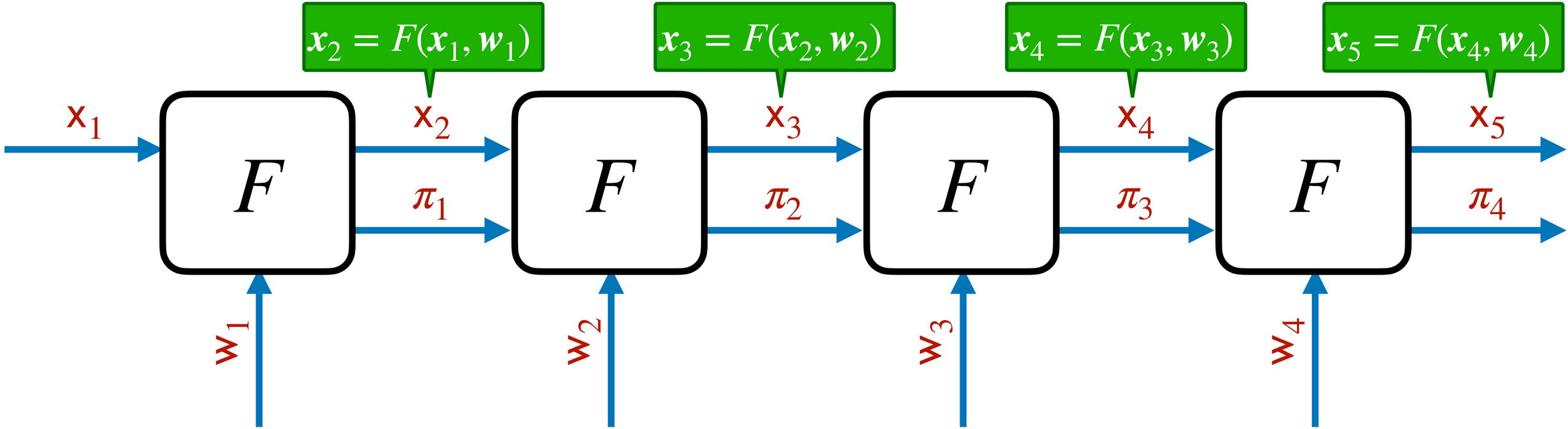
Incrementally Verifiable Computation



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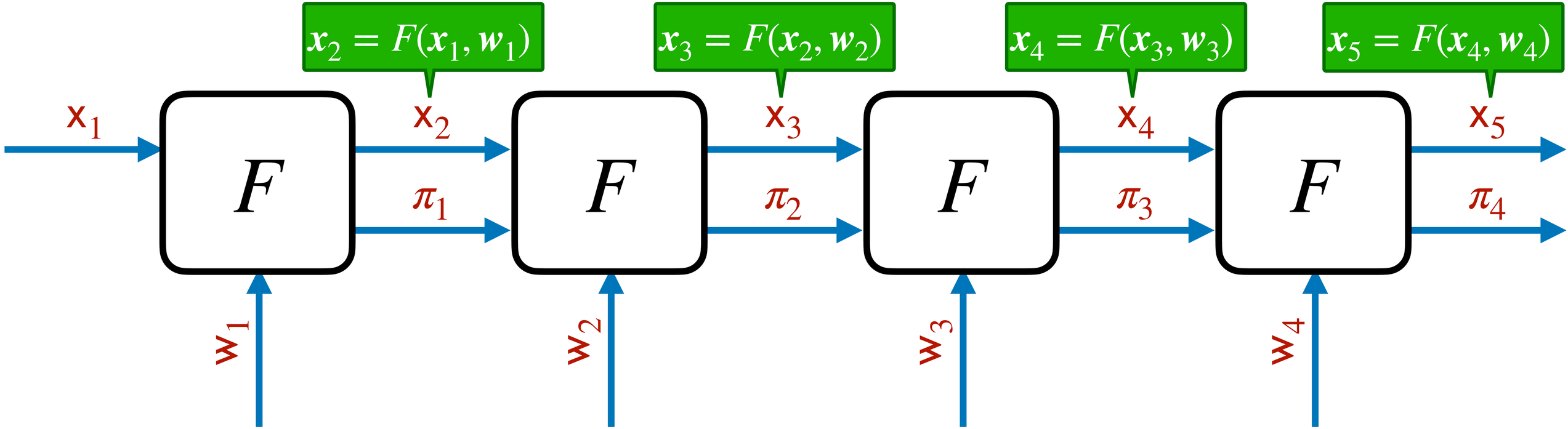

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Current Machinery

The Current Machinery of ZK-SNARKs

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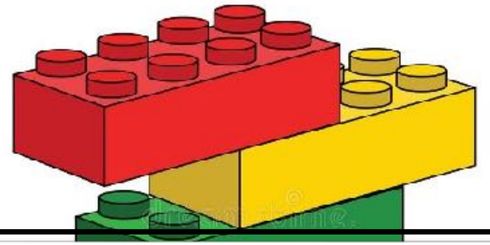
Intermediate Representation

DSL



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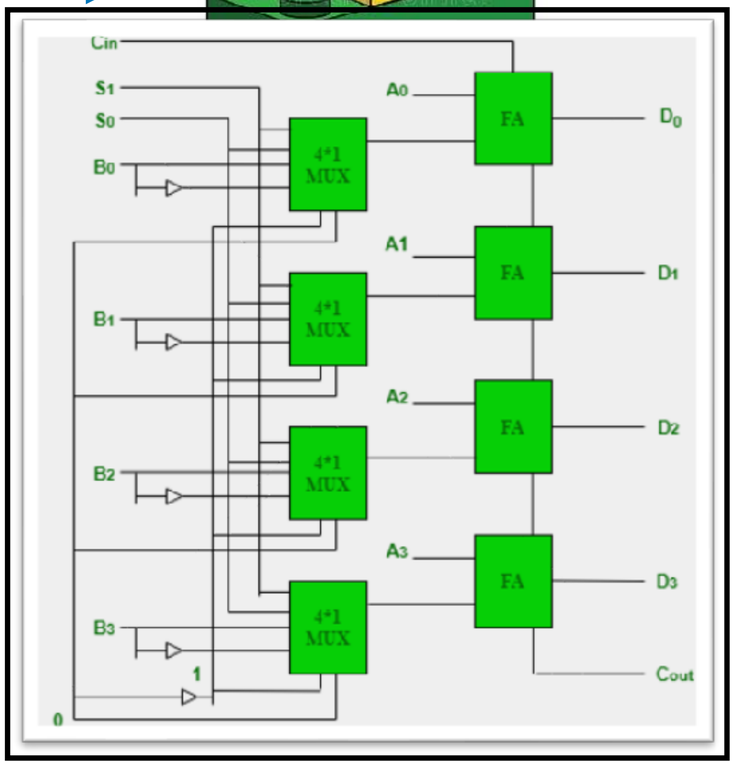

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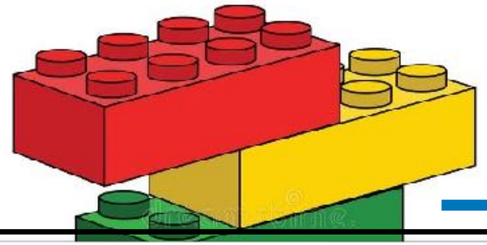

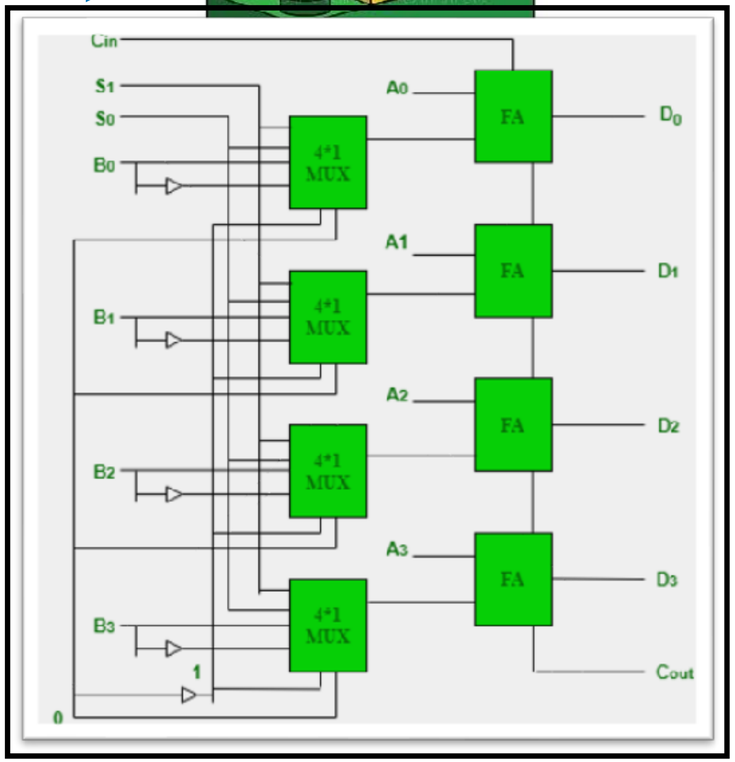

Cryptography

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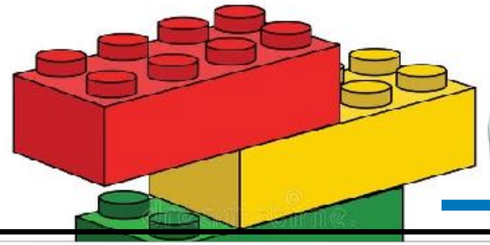

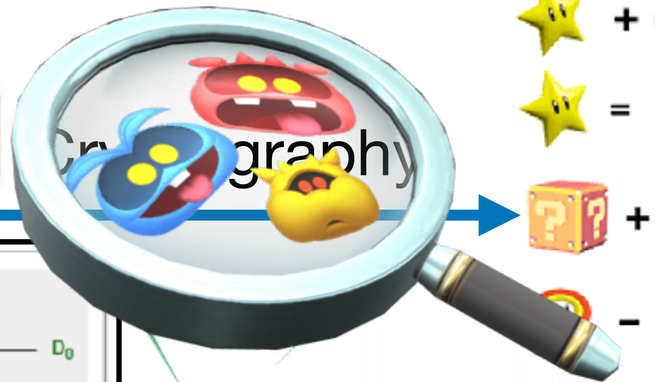


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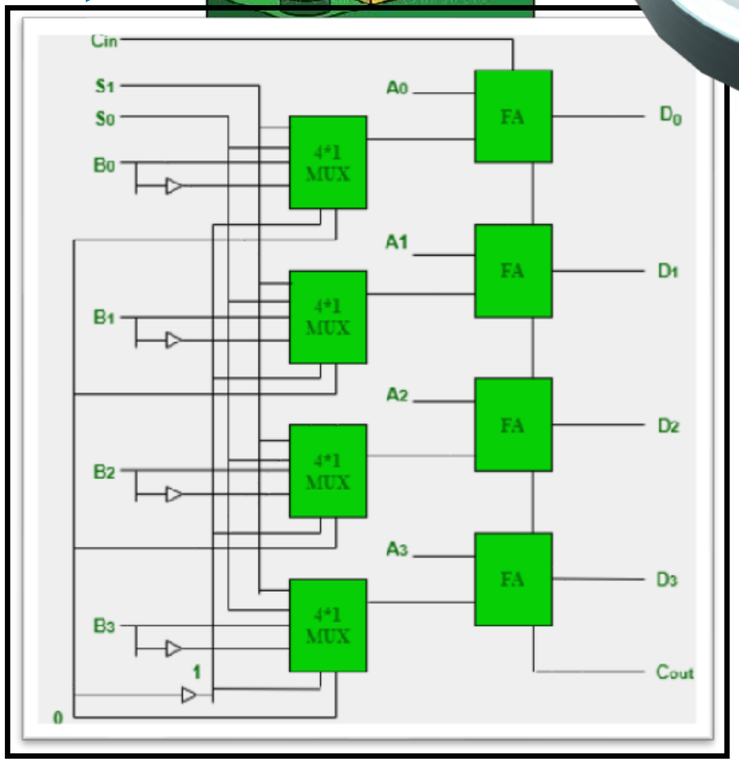

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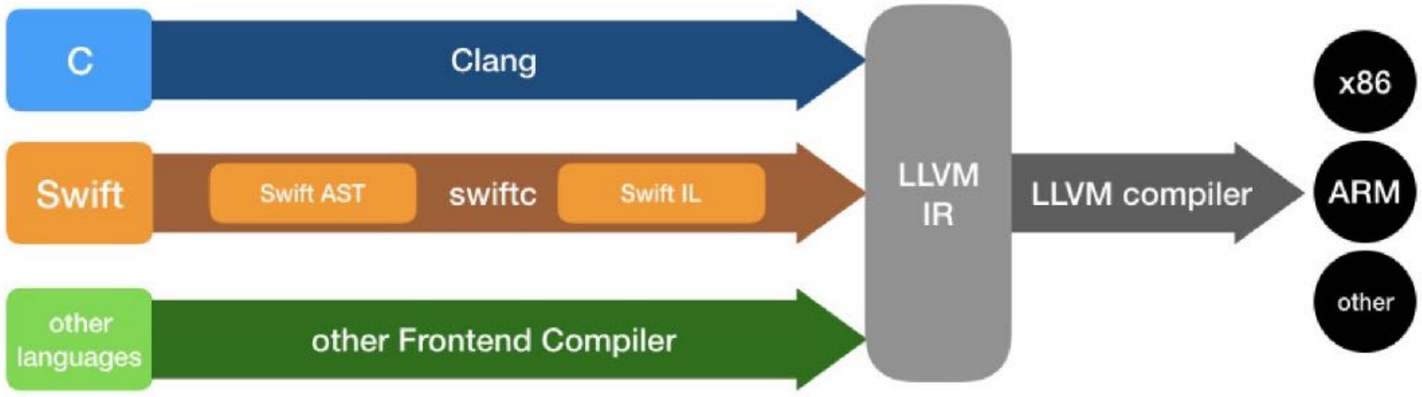

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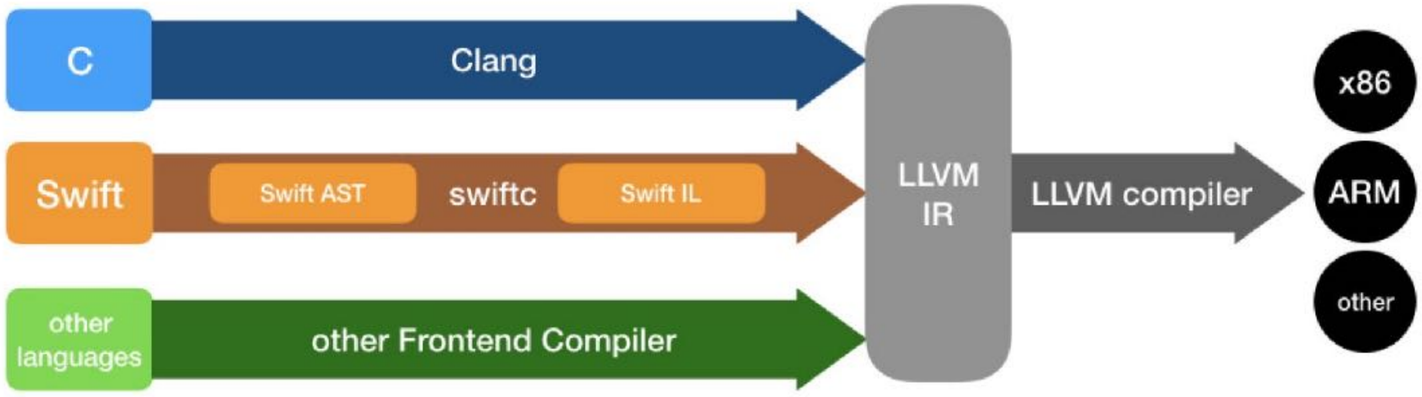

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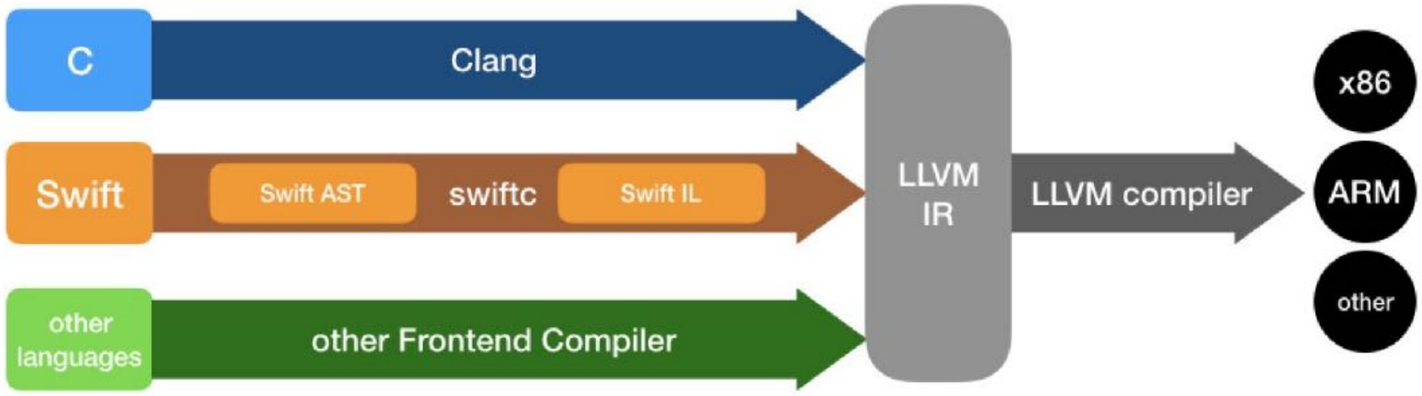

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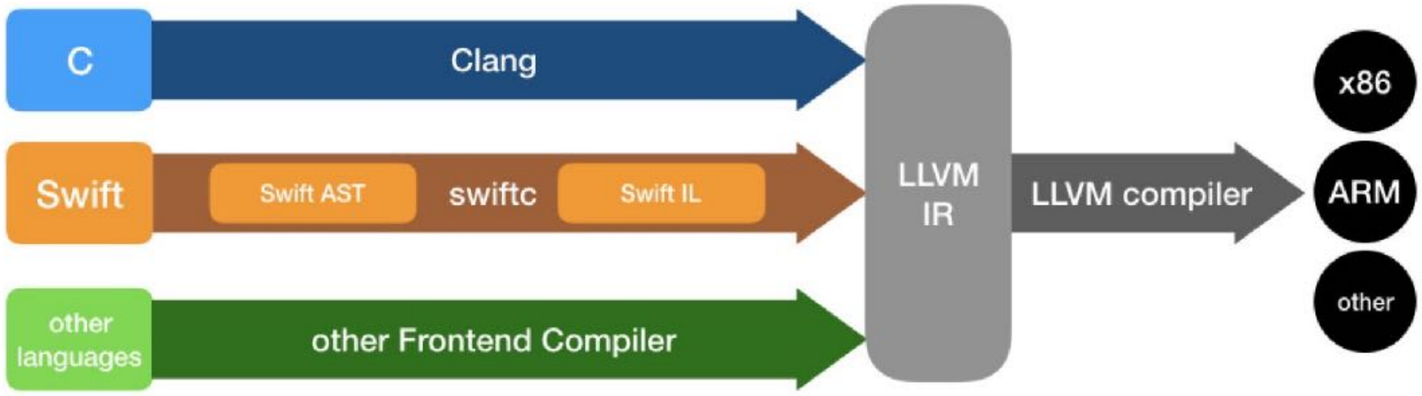

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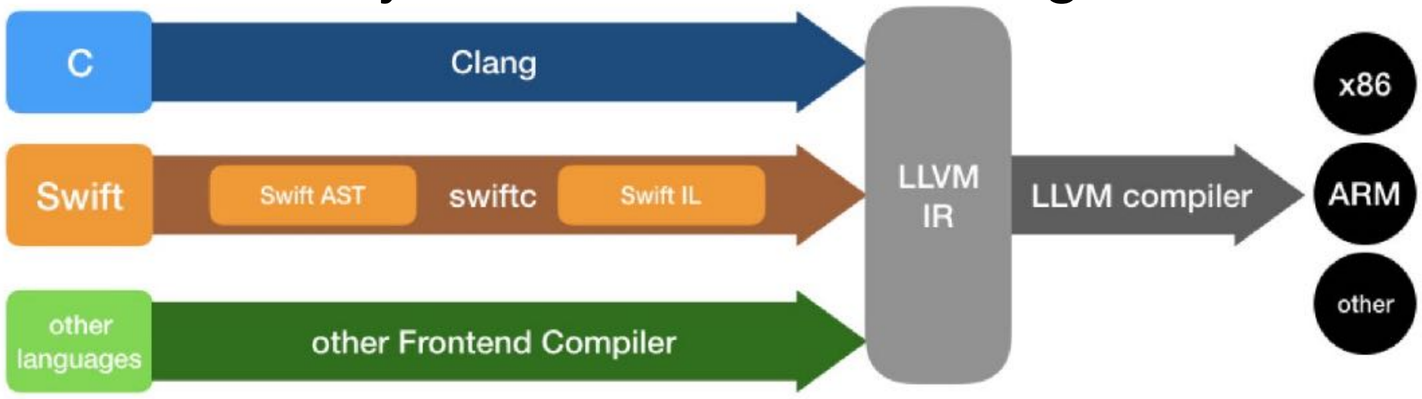

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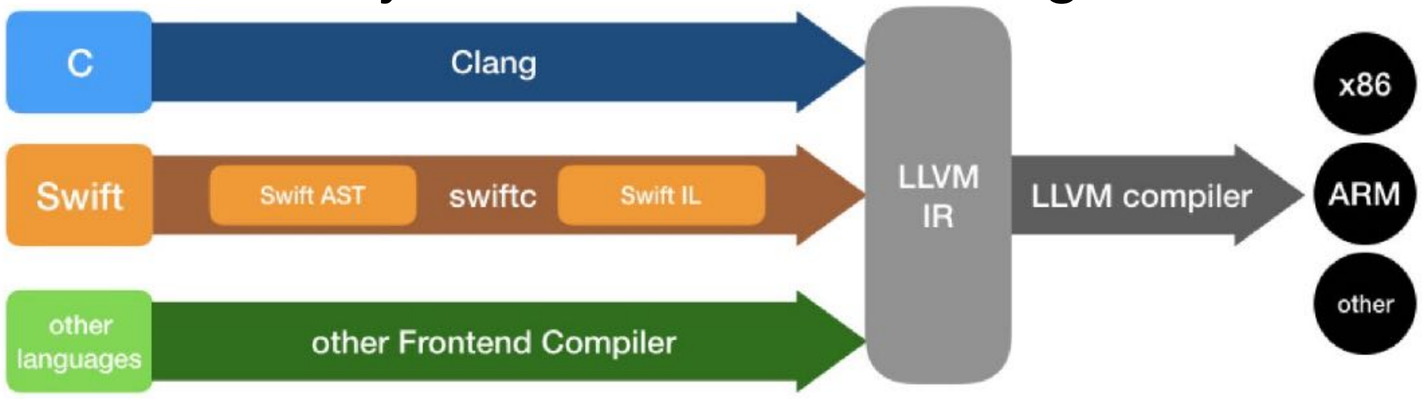

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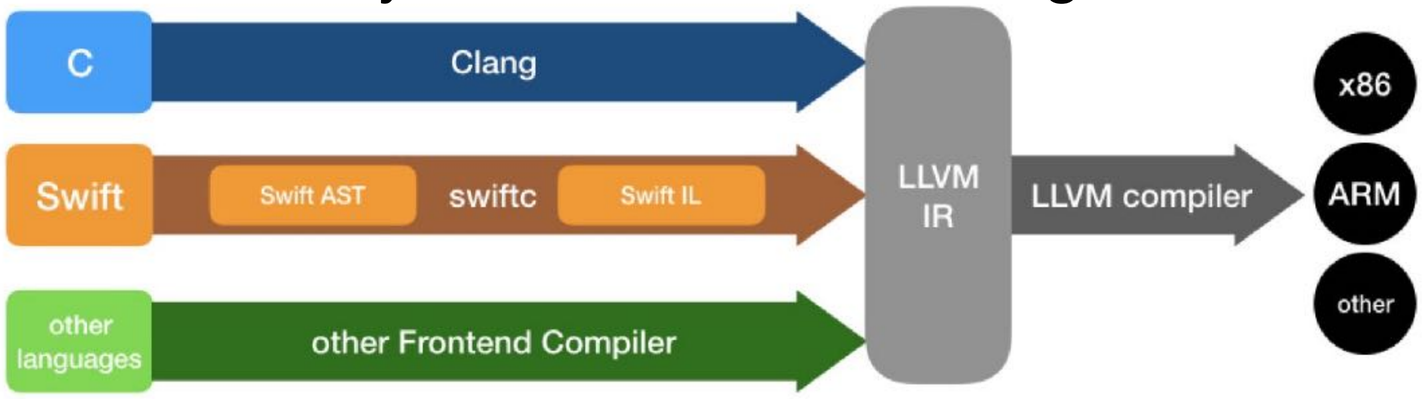

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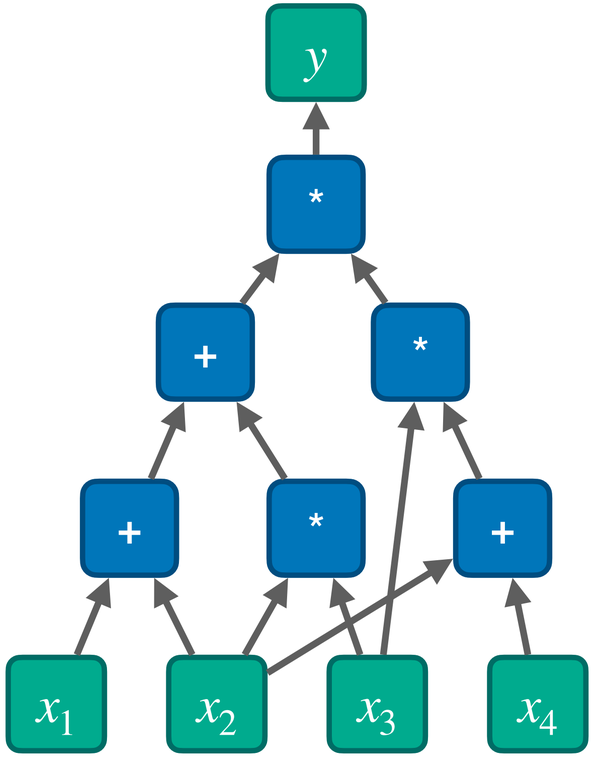

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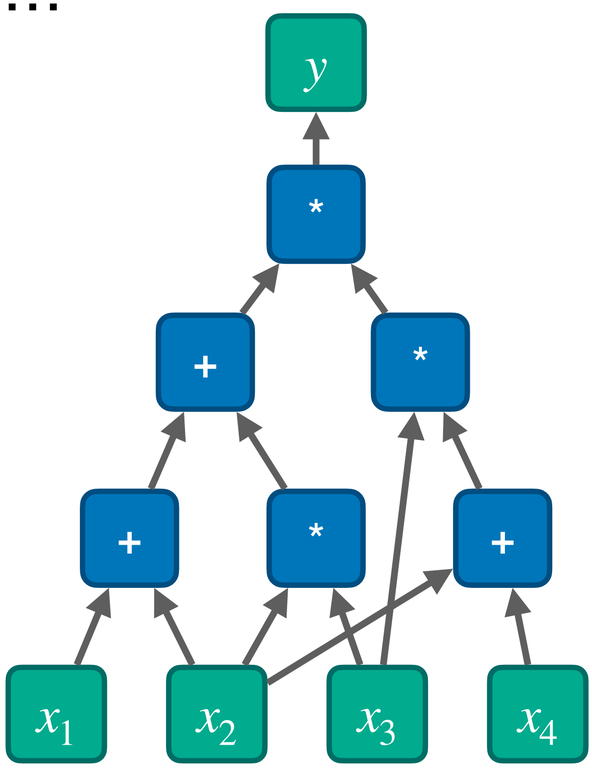

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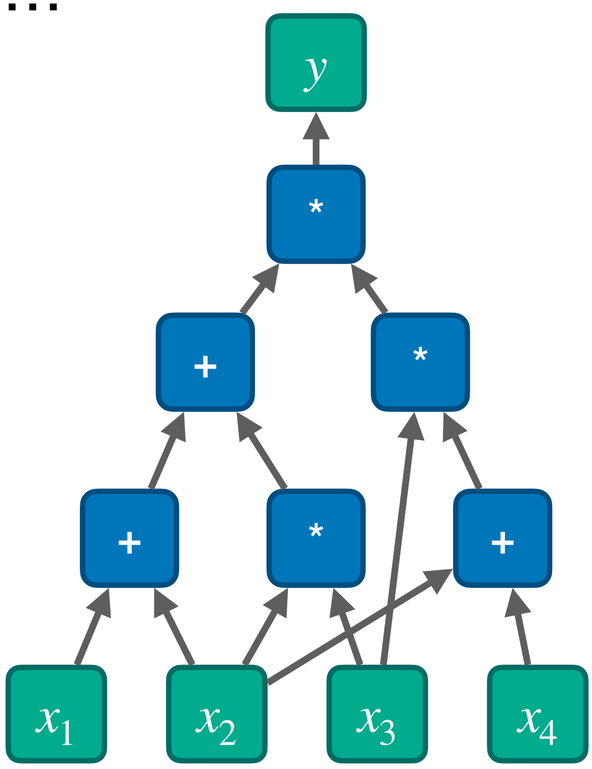

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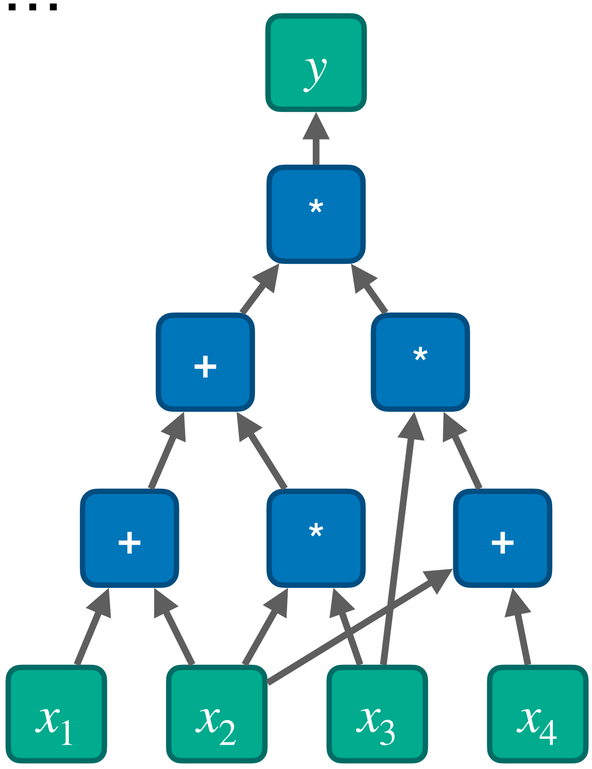

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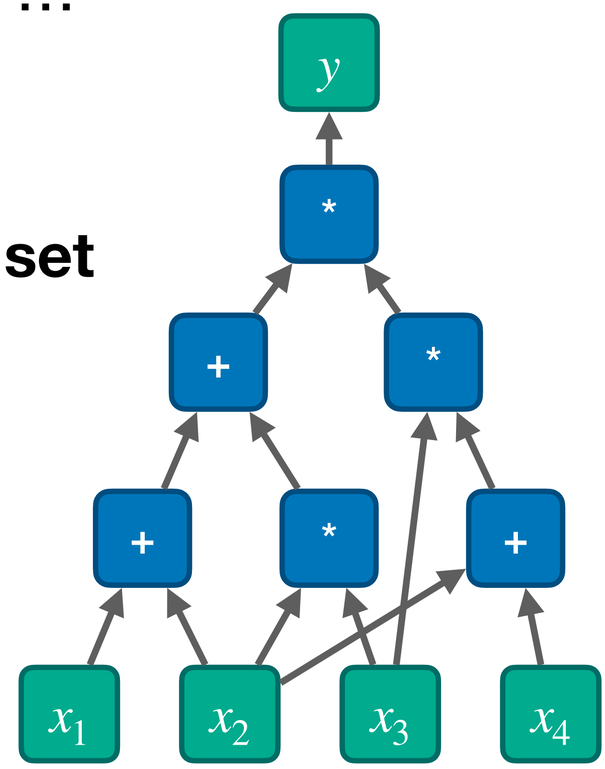

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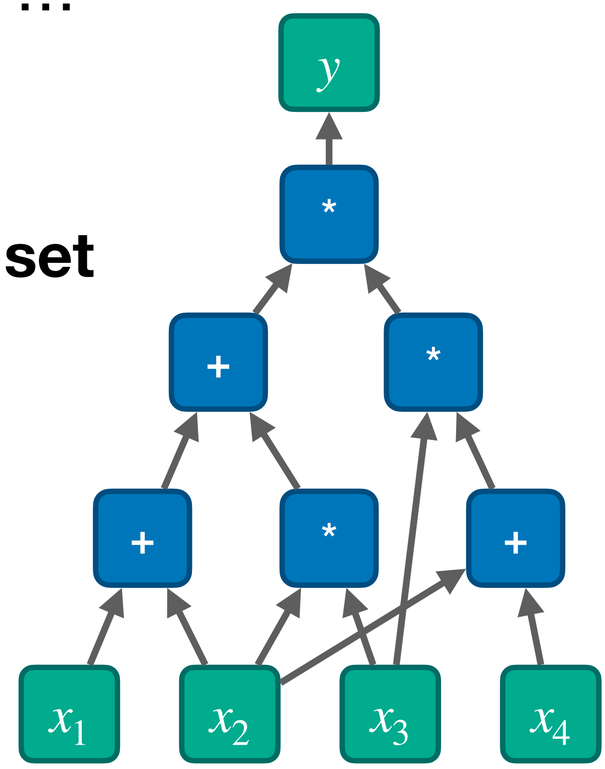

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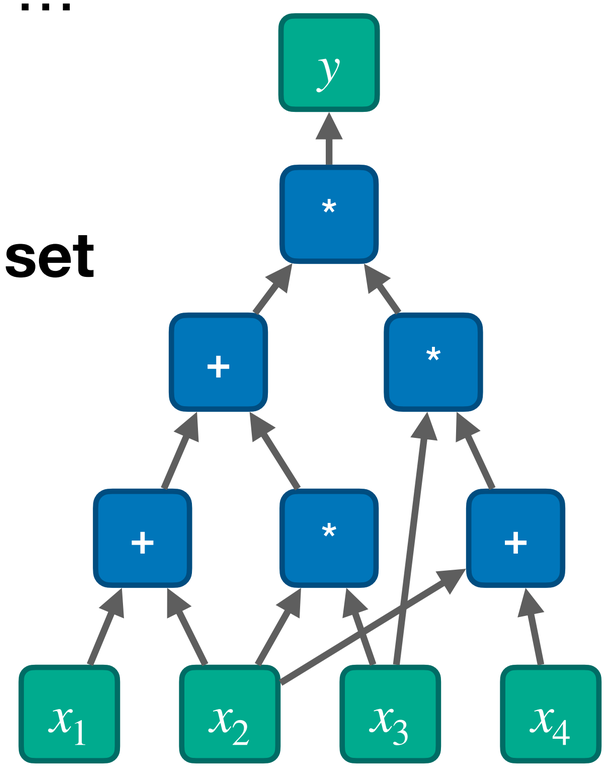

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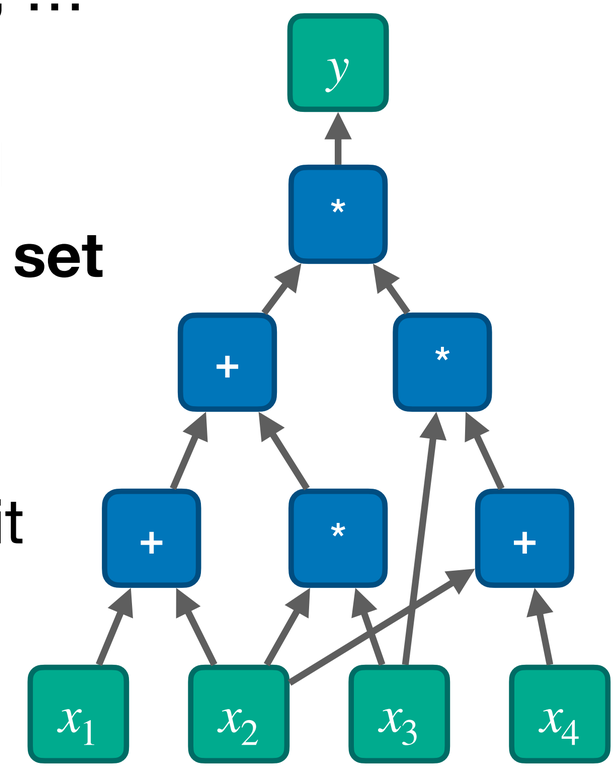

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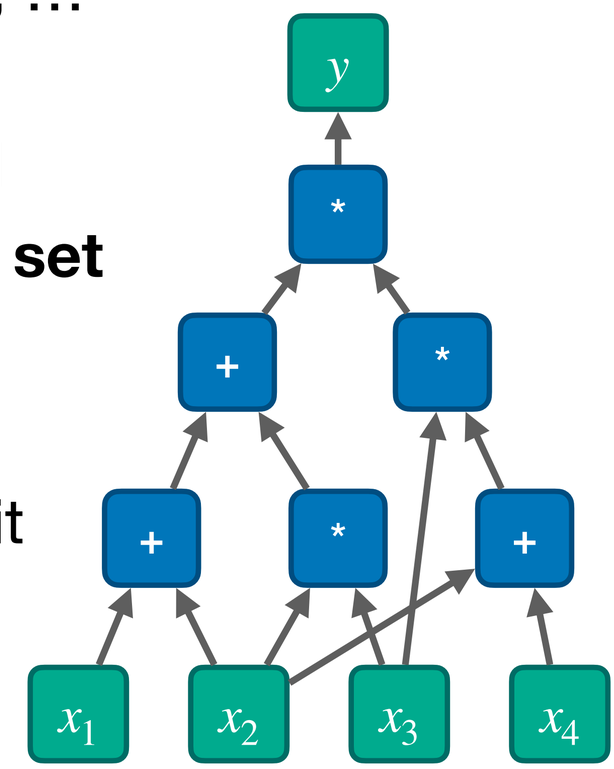

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  - **Optimization** depends on IR; certain operations are “unexpectedly” costly
- DSLs change a lot, but are fun
  - Learn one: then you see how big the intermediate interpretations can be
    - And how efficient a SNARK you need

On Domain-Specific Languages

```
#![no_std]
#![no_main]

fn fib(n: u32) -> u32 {
    match n {
        0 => 0,
        1 => 1,
        _ => fib(n - 1) + fib(n - 2),
    }
}

#[nexus::main]
fn main() {
    let n = 7;
    let result = fib(n);
    assert_eq!(result, 21);
}
```

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- Many DSLs by now
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On Domain-Specific Languag

- Collaboration questions:
- How to compile high-level code efficiently to ZK IRs?
- How to formally verify that compilation was correct?
- Design etc of DSLs

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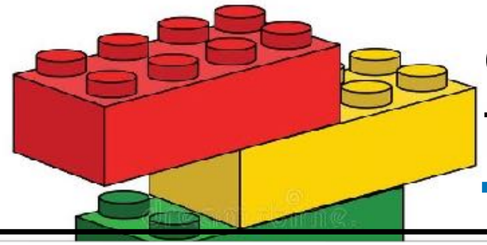
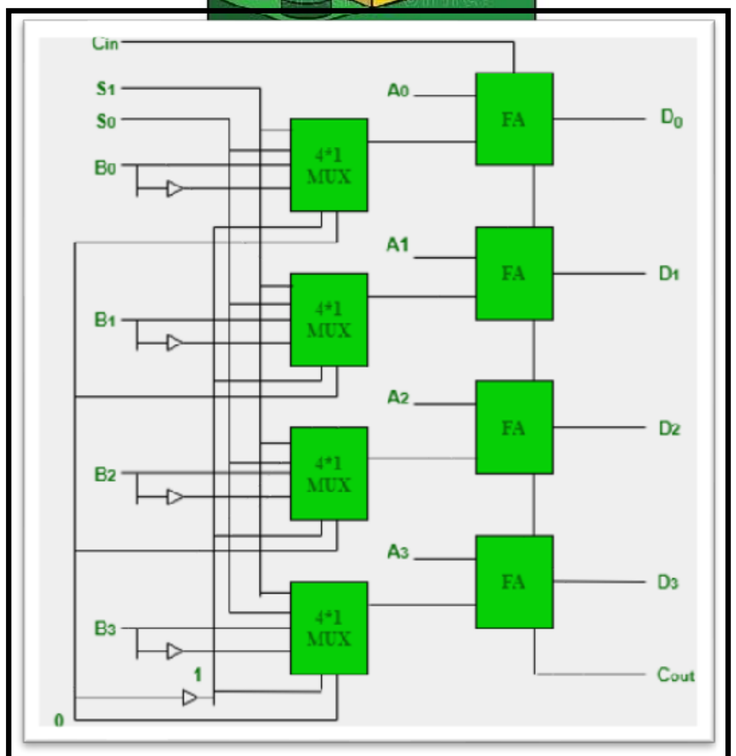
Backend

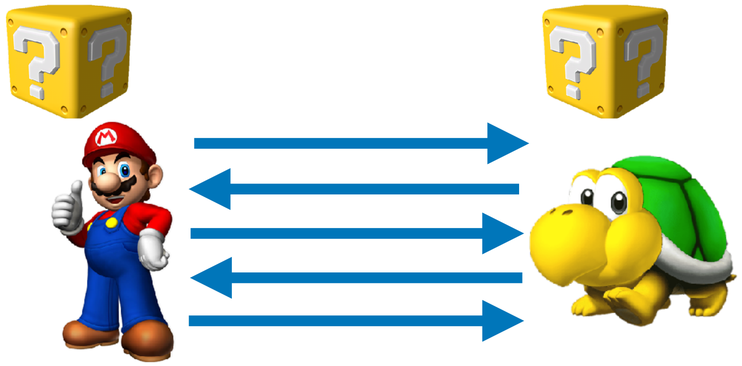
Intermediate Representation

(Polynomial) Interactive Oracle Proof



"Non-cryptographic" techniques



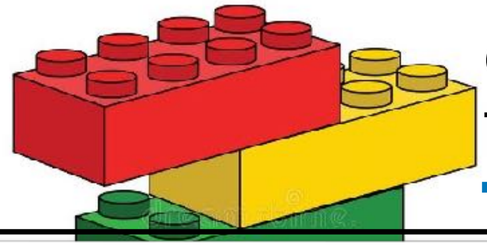


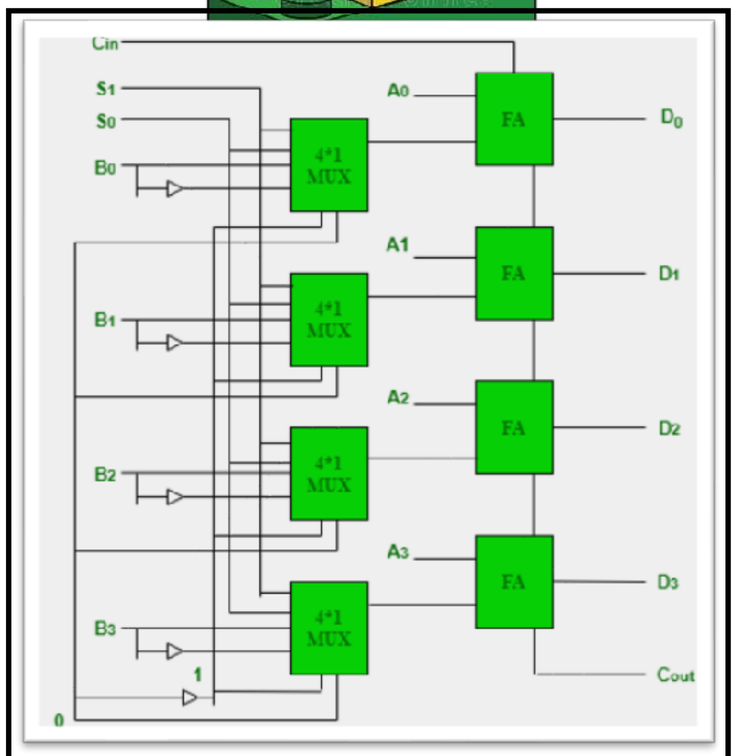
Backend

Intermediate Representation

(Polynomial) Interactive Oracle Proof

ZK-SNARKs





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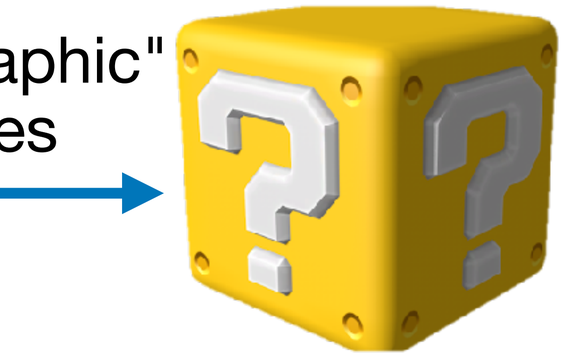
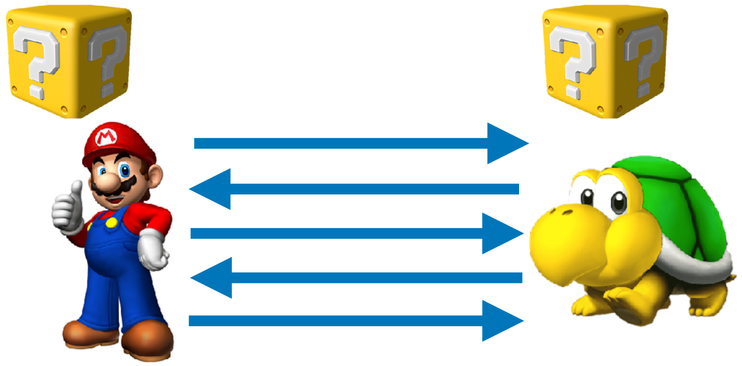
Cryptography





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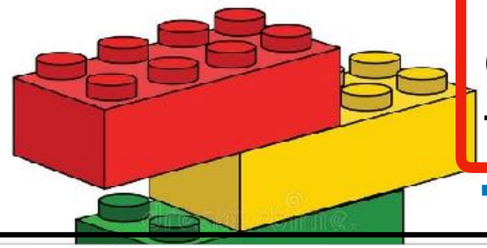

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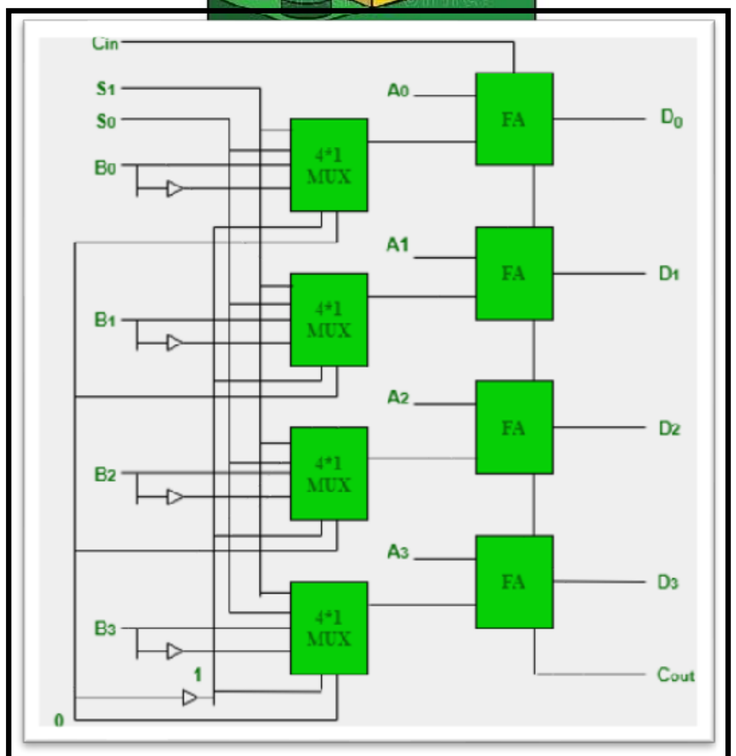

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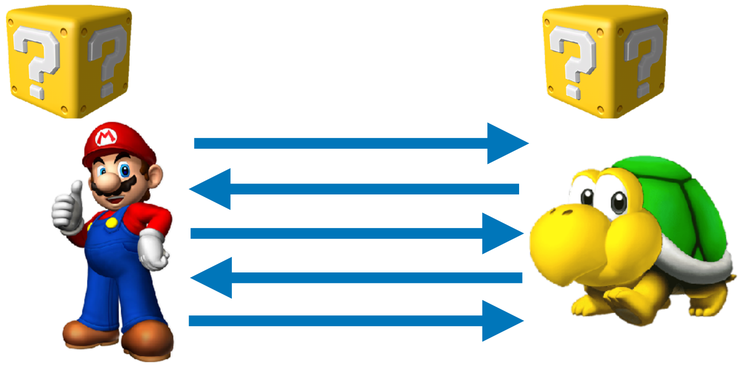

Usual Proof

$x, w = \boldsymbol{a} \in \mathbb{F}^n$

x

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Read every bit
Accept/reject

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NP: class of languages that have proofs w which can be verified in polynomial time

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Usual Proof

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NP: class of languages that have proofs w which can be verified in polynomial time

Think of $n = 2^{30}$

Even prover time $n^2 = 2^{60}$ is impenetrable

We want verifier to be much faster than 2^{30}!

Read every bit
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Probabilistically Checkable Proof (~1992)

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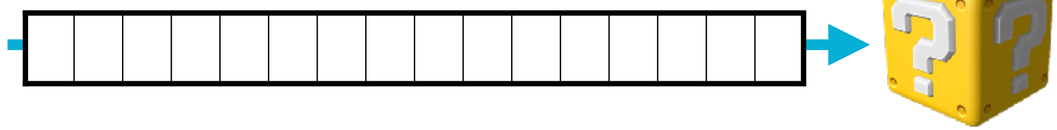

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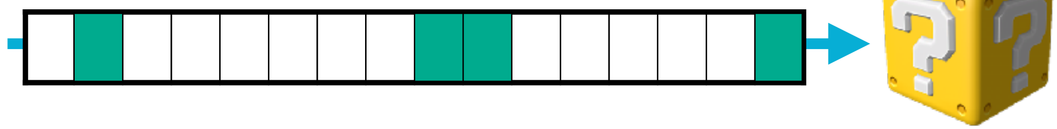

V can toss random coins (not secure if V is deterministic)

$i \in [1, \ell(n)]$

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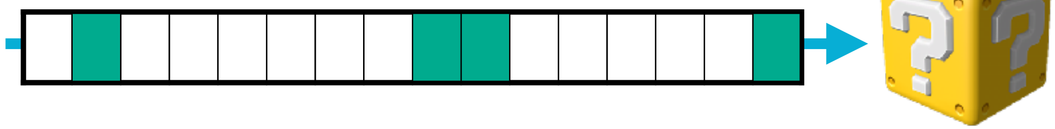

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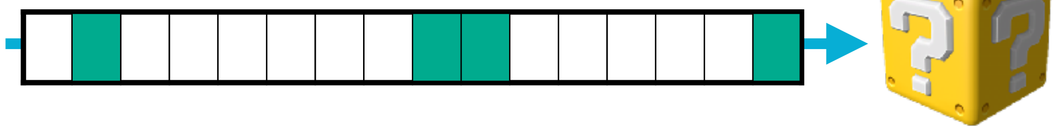

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Key insight: allowing <u>randomness</u> makes it much more efficient to <u>verify</u>!

PCP Theorem

$x, w = a \in \mathbb{F}^n$

$b = \mathrm{Enc}(a) \in \mathbb{F}^{\ell(n)}$

PCP$[r(n), q(n)]$ - class of languages where proofs can be verified by using $r(n)$ verifier's random bits and $q(n)$ queries

PCP theorem: PCP$[O(\log n), O(1)] = \mathrm{NP}$

- Celebrated as one of the most central theorems in complexity theory

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😭😭😭😭 Known PCPs are quite inefficient for the prover (proof length $O(n \log^4 n)$)

Accept/reject based on queried bits

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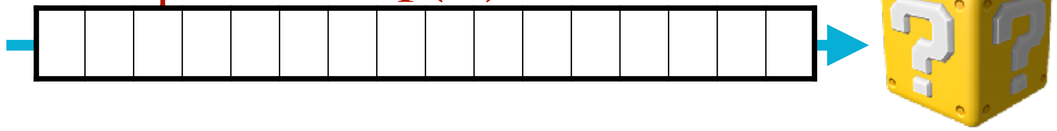

$r_1 \in \mathbb{F}$

Random, unpredictable, independent from previous messages

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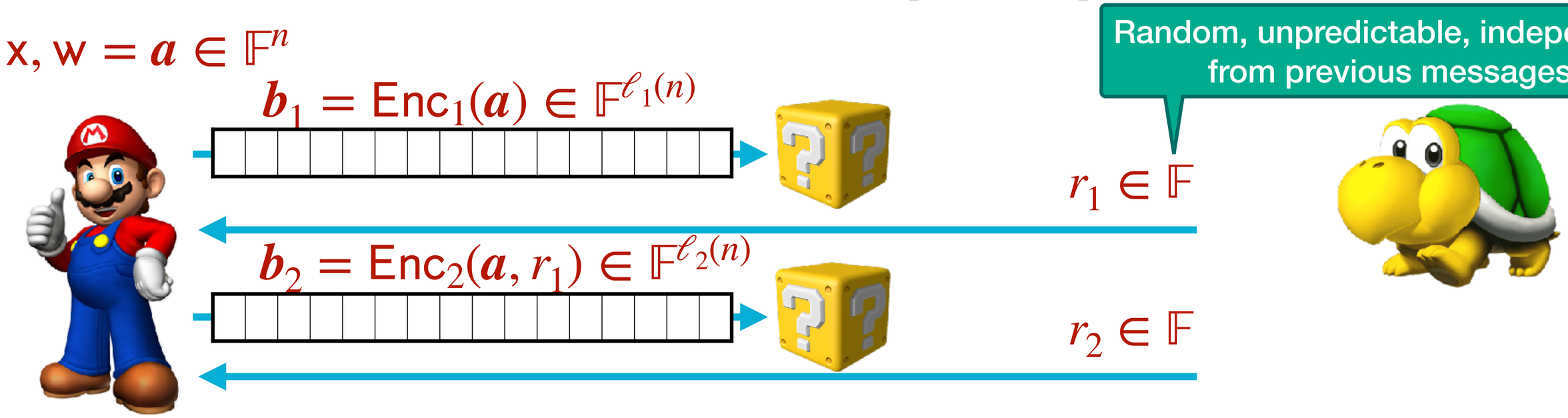

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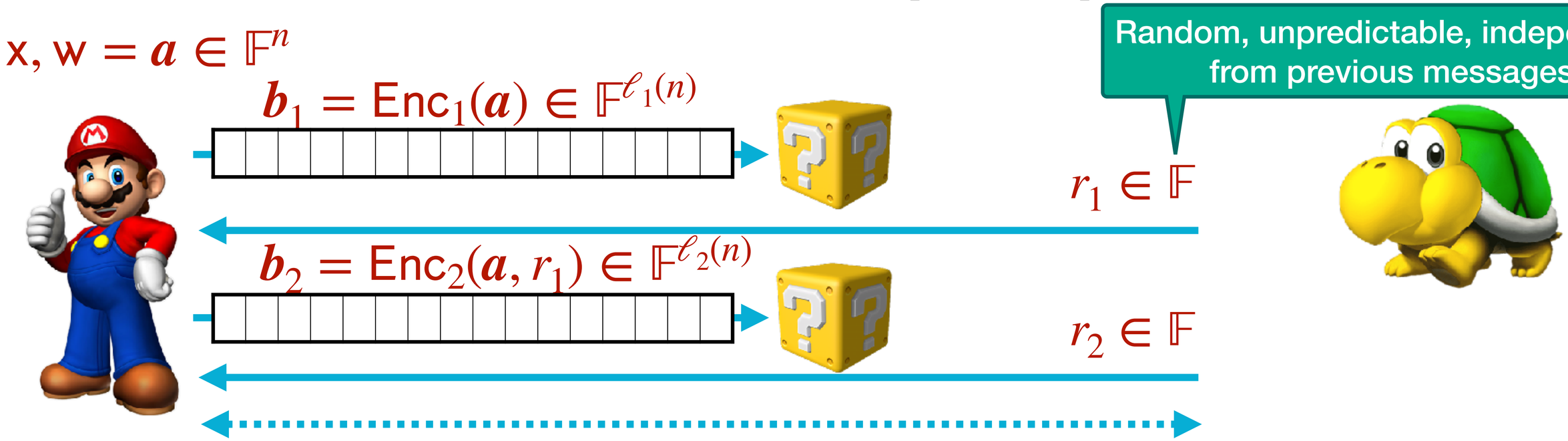

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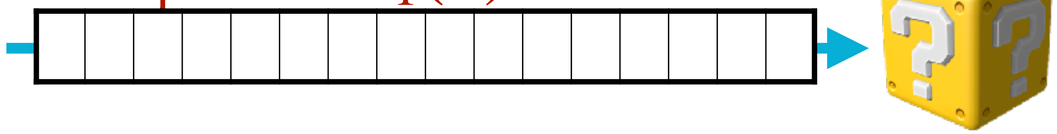

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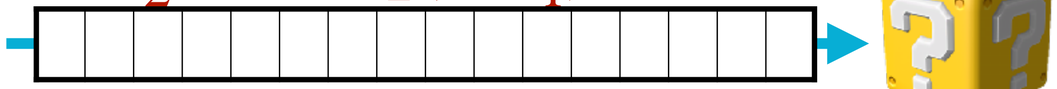

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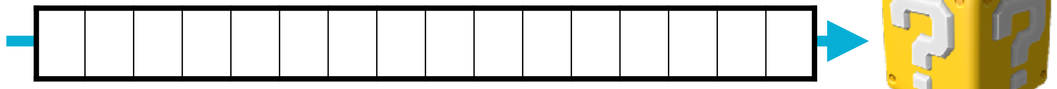

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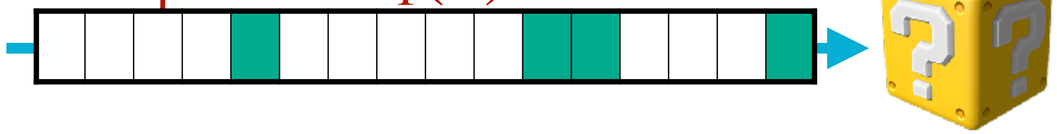

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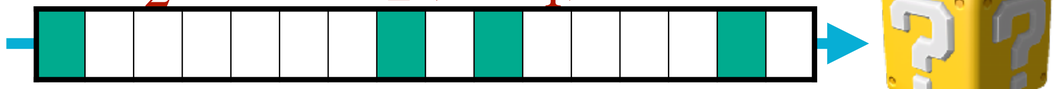

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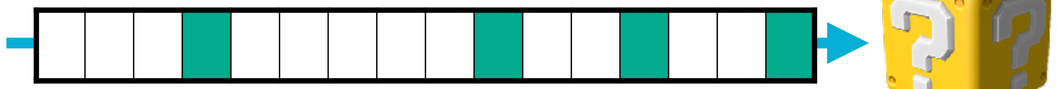

i, j

$b_j[i]$

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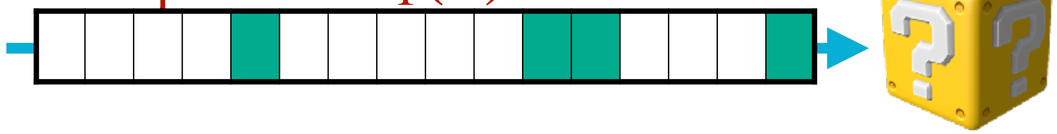

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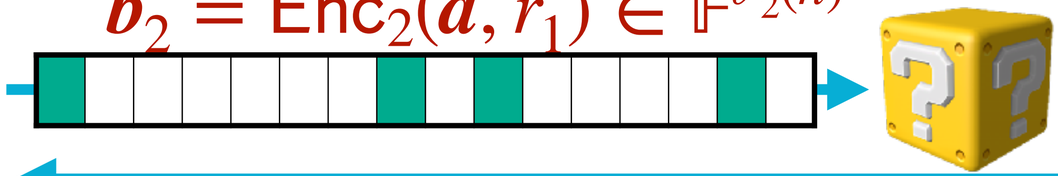

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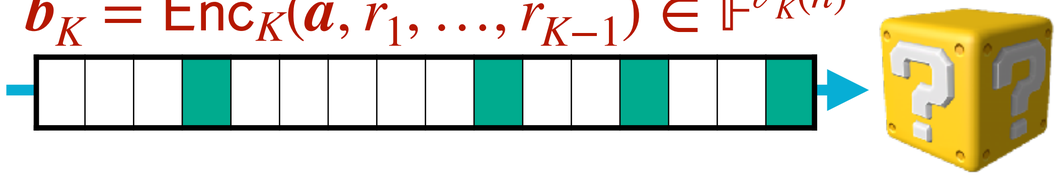

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Random, unpredictable, independent from previous messages

Accept/reject based on randomizers and queried bits

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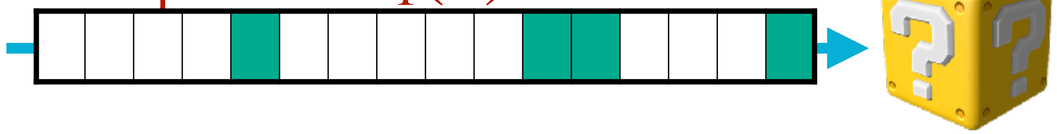

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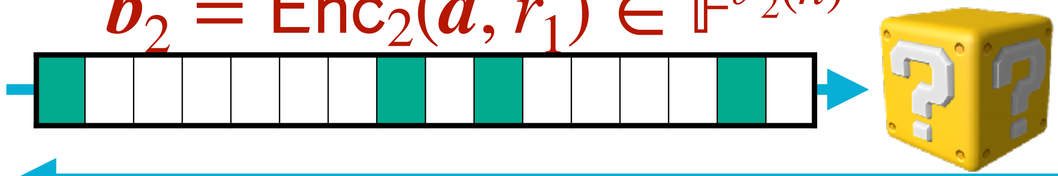

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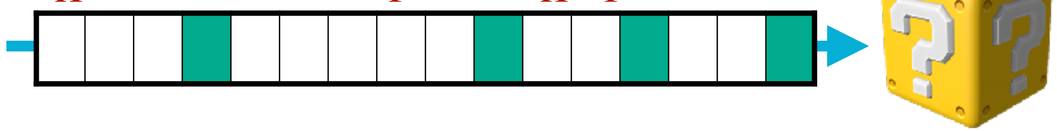

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Key insight (1980+): allowing interaction makes it much more efficient to prove and verify

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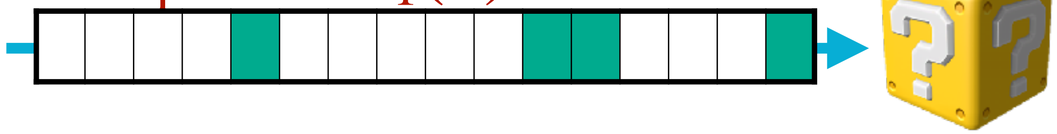

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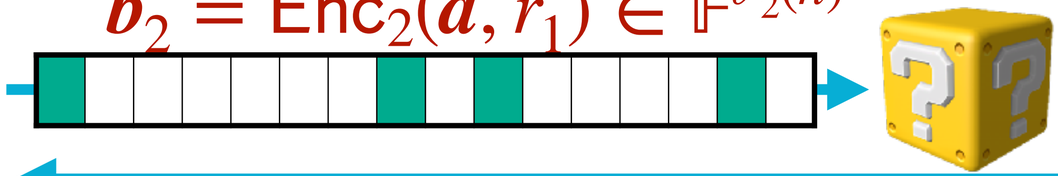

$r_2 \in \mathbb{F}$

$b_K = \mathsf{E}$

Random, unpredictable, independent from previous messages

Key ideas:

- V can spot-check certain coordinates of the encodings
- For efficiency, need a tool to "**smear**" errors
- => Use error-correcting codes

Key insight: <u>interaction</u> makes it much more efficient to <u>prove</u> and <u>verify</u>

$b_j[i]$

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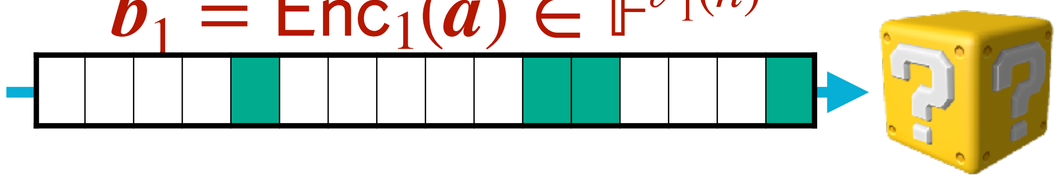
Interactive Oracle Proof (

- Collaboration questions:
- Find good error-correcting codes
- Find out which codes are needed…
- Give input to cryptographers about known coding theory facts

$x, w = a \in \mathbb{F}^n$

$b_1 = \mathrm{Enc}_1(a) \in \mathbb{F}^{\ell_1(n)}$



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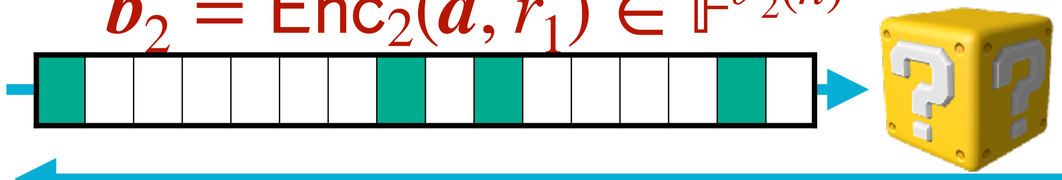

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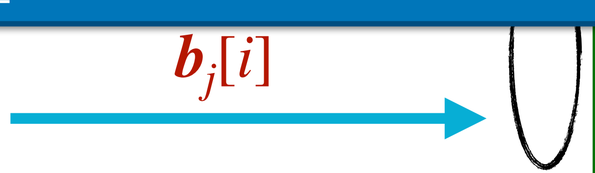

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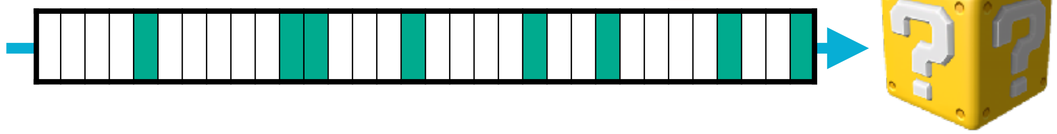
Polynomial Interactive Oracle Proof (2020)

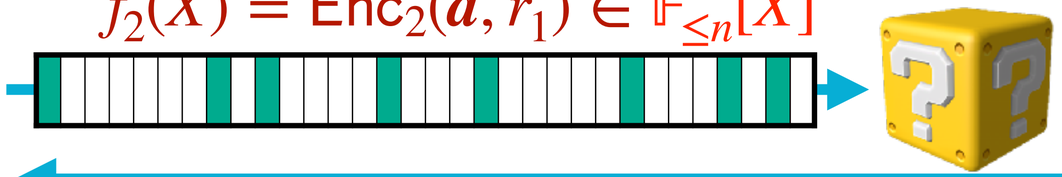
$x, w = \boldsymbol{a} \in \mathbb{F}^n$

$f_1(X) = \mathsf{Enc}_1(\boldsymbol{a}) \in \mathbb{F}_{\leq n}[X]$



$r_1 \in \mathbb{F}$

$f_2(X) = \mathsf{Enc}_2(\boldsymbol{a}, r_1) \in \mathbb{F}_{\leq n}[X]$



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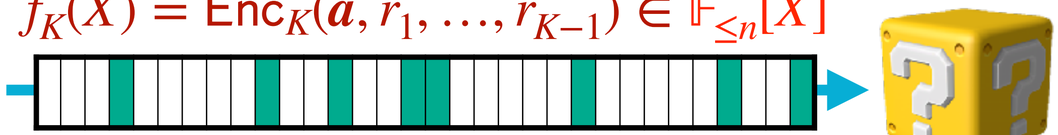

i, j

$f_j(i)$

Random, unpredictable, independent from previous messages

Accept/reject based on randomizers and queried evaluations

Polynomial Interactive Oracle Proof (2020)

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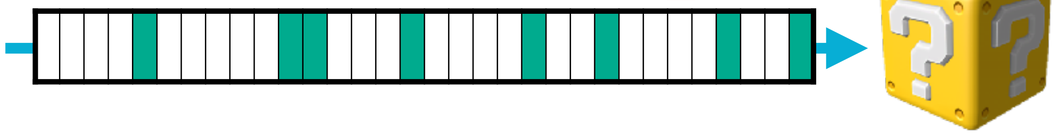

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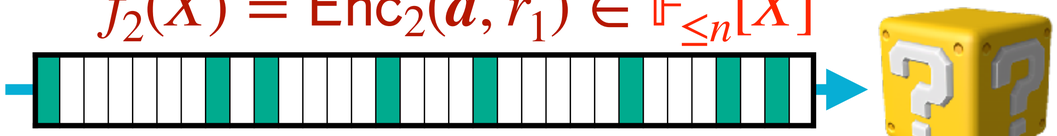

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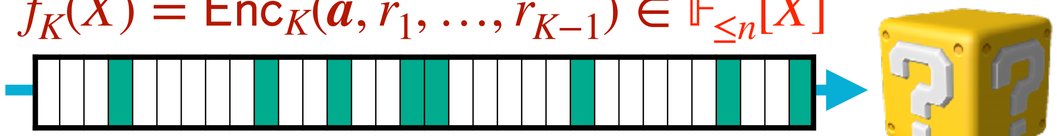

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$|\mathbb{F}|$ possible queries i instead of just n

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Accept/reject based on randomizers and queried evaluations

Key insight (trade-off): allowing a more powerful oracle makes IOP more efficient but implementing the oracle is more costly

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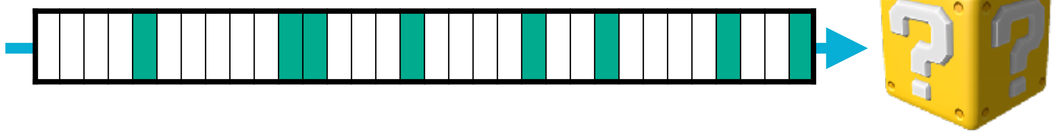

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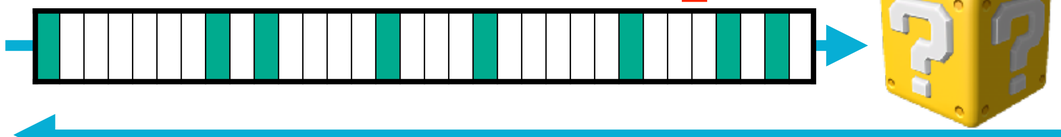

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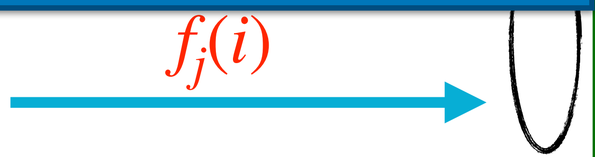

s i instead of just n

Accept/reject based on randomizers and queried evaluations

Key ideas:

- V can query polynomials at random points
- Since low-degree polynomials have few roots, random evaluation = 0 $\Longrightarrow$ polynomial = 0 w.h.p

Key insight (tra… more powerful … more efficient but implementing the oracle is more costly

Insufficiency

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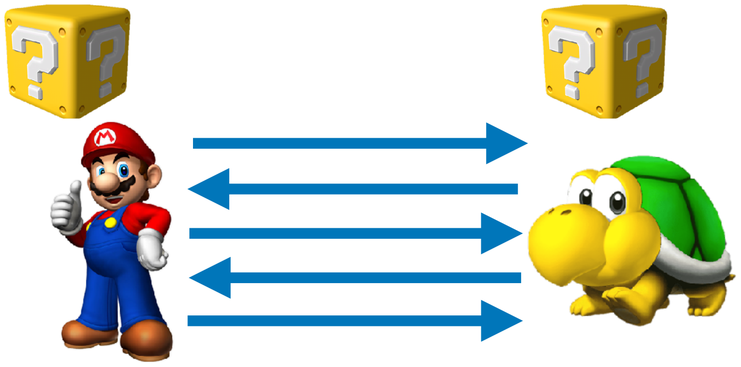
  - … and also **only** computationally secure

Backend

(Polynomial) Interactive Oracle Proof



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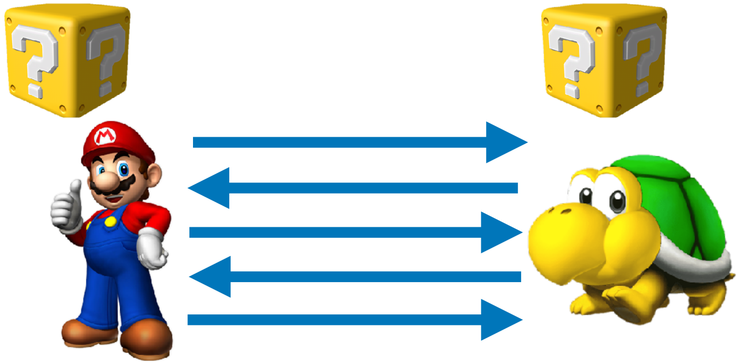
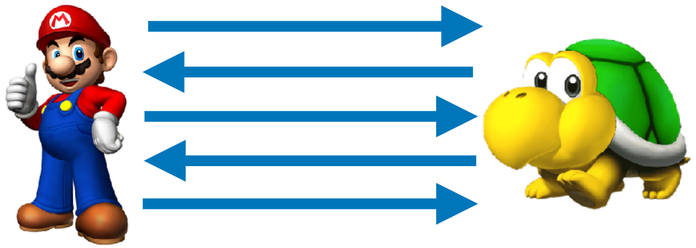
(Polynomial) Interactive Oracle Proof

Interactive Protocols

Commitment





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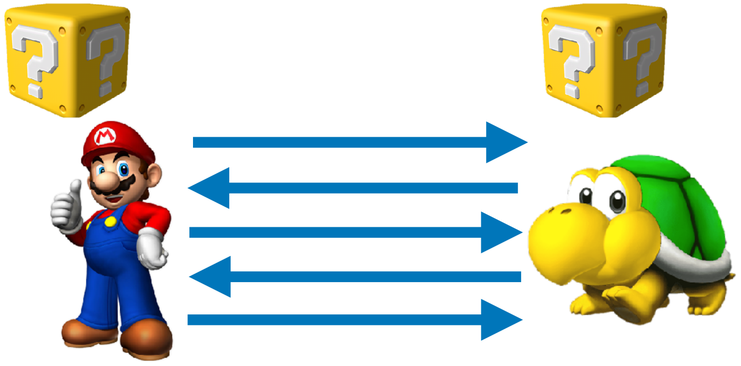
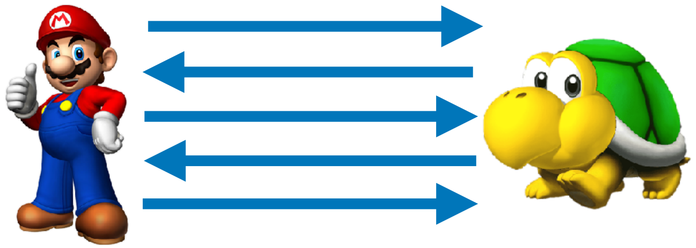
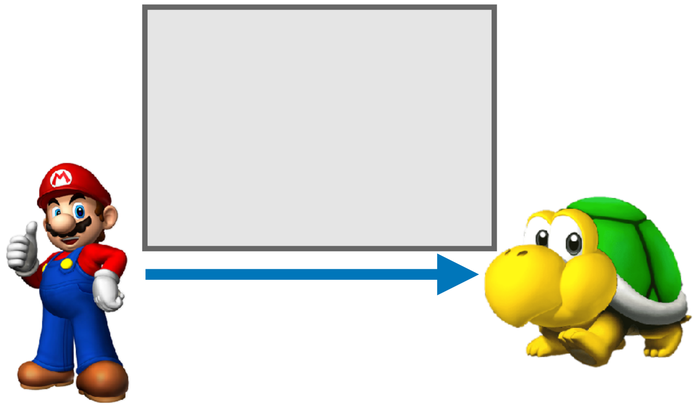
Fiat-Shamir

ZK-SNARKs







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We will not discuss FS today

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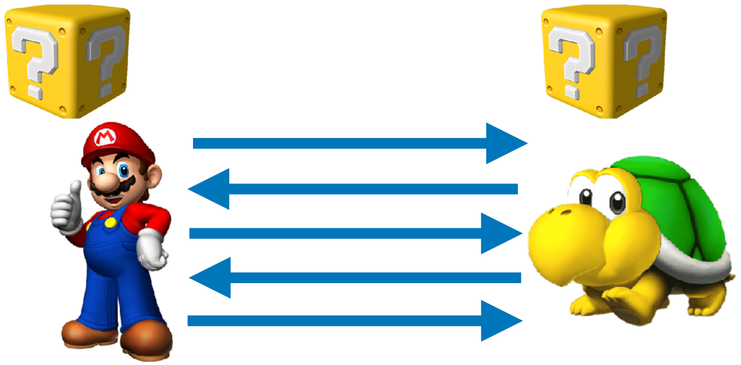

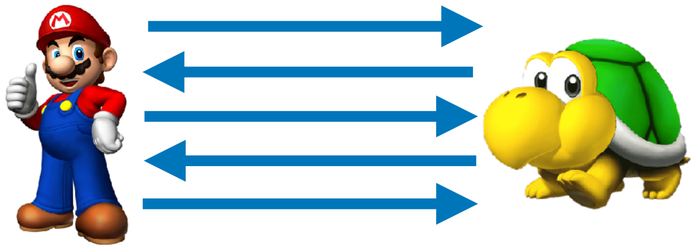

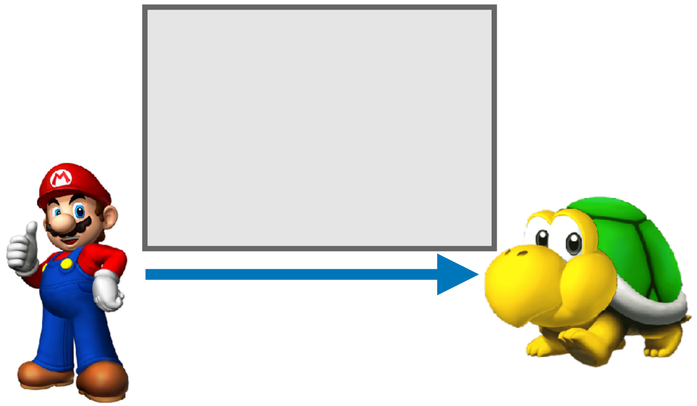

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Security assumptions and trusted parameters depend only on the VCS/PCS

Current Main "Cryptographic" Approaches

	Assumption	Post-Quantum	Trusted para-meters	Prover speed	Verifier speed	Argument length	Examples
Pairing-based PCS + PIOP	Various pairing-based (elliptic curve) assumptions	No	Yes	O(n log n) heavy operations	Small constant number of heavy operations (a few milliseconds)	Very short (<600 B for any computation)	Plonk, Marlin, Groth16, Polymath, Lunar, Basilisk, Vampire, Spartan, HyperPlonk, BabySpartan, ….
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  - Just search for anything (for example, “GKR")

Background Material

- **Thousands of YouTube videos**
  - Just search for anything (for example, “GKR")
  - But ask me for who the good presenters are

Important References

- (Usual proofs:) Euclid. *Elements* (~300 BC)
- PCP:
  - Sanjeev Arora, Shmuel Safra. **Probabilistic checking of proofs: A new characterization of NP** (1998)
  - Sanjeev Arora, Carsten Lund, Rajeev Motwani, Madhu Sudan, Mario Szegedy. **Proof verification and the hardness of approximation problems** (1998)
- IOP:
  - Eli Ben-Sasson, Alessandro Chiesa, Nicholas Spooner. **Interactive Oracle Proofs** (2016)
  - Omer Reingold, Guy N. Rothblum, Ron D. Rothblum. **Constant-round interactive proofs for delegating computation** (2016)

Important References

- PIOP:
  - Benedikt Bünz, Ben Fisch, Alan Szepieniec. **Transparent SNARKs from DARK compilers** (2020)
  - Alessandro Chiesa, Yuncong Hu, Mary Maller, Pratyush Mishra, Noah Vesely, Nicholas Ward. **Marlin: Preprocessing zkSNARKs with Universal and Updatable SRS** (2020)
- PCS:
  - Aniket Kate, Gregory M. Zaverucha, Ian Goldberg. **Constant-Size Commitments to Polynomials and Their Applications** (2010)
- (Definition of interactive protocols and ZK) Shafi Goldwasser, Silvio Micali, Charles Rackoff: **The Knowledge Complexity of Interactive Proof-Systems.** STOC 1985: 291-304
- Noir: https://noir-lang.org/